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**ESTUDO DAS EMOÇÕES COM TEORIA DAS CATEGORIAS
EMOTION STUDY USING CATEGORY THEORY**

por

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ABSTRACT

This work is studying two emotion theories using category theory. It studies these theories using the notion of Appraisal, a central concept in these theories. To implement the notions of category theory it is used some mathematical algorithms called: Pearson Coefficient, Principal Component Analysis and K-means clustering. Pearson Coefficient is used to observe if there is a linear relation between emotions. Principal Component Analysis is used to reduce the dimensionality of vectors (representing emotions) to observe similar variation in emotions. Finally, k-means provides clustering between emotions with similar variation. Once the clusters are formed it is used category theory to provide the connection between emotions conserving the structure of the theories. One theory of emotions is *The Cognitive Structure of Emotions* of Andrew Ortony, Gerard L. Clore and Allan Collins; the other one is Klaus Scherer's theory of emotion *Sequential check theory of emotion differentiation*. Due to the complexity of the theories it is proposed a category theory model for specific parts of each theory. It is proposed category theory as an option to establish connections between theories conserving the structure of them because in the state of the art there is not a development to provide connections between emotion theories.

Keywords: Appraisal - Emotion Theories, Category Theory, Principal Component Analysis

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1 INTRODUCTION

Emotion study has more than 2500 years of history, from Greek philosophers as Aristotle (thinking emotions as reaction (can be pleasure or pain) to people actions (Plamper, 2015), passing through thinkers as Descartes, Spinoza, Hobbes, Hume, Rousseau, Kant, just to mention a few and a rapid exposition of their thinking about emotions. Descartes positions with respect to emotions is actually discussed, however, one point of view is that he considers emotions like been part just of the body and not been part of the mind (Plamper, 2015). So, this point of view inspire to the artist from century XVII Charles Le Brun to capture the emotions in the body, in the face, then, Charles Le Brun can be consider the forerunner of image recognition of emotions (Plamper, 2015) (see Figure 1.1).



Figure 1.1: Emotions: Admiration and Fear by Charles le Brun (le Brun, 1732)

Spinoza's point of view is that emotions are in body and mind because he considers that body and mind are two faces of the same reality. Spinoza thought is study in neuroscience (Plamper, 2015). Hobbes view about emotion considers that there is a natural state that is a living-out of passion and it is regulated in the society through fear, balance between passions and fear regulate human being and let think (reflect) (Plamper, 2015). Hume considers that passions control reason, but he introduce the notion of *sympathy*, currently, a notion used and study. Rousseau considers that the human being is full of passion and culture corrupt his nature that defines all humans as equals. With the beginning of Enlightenment reason and emotion are divided, this view is supported by Kant who considers emotion as oposed to reason (Plamper, 2015).

Emotion study is divided in two groups: *Social Constructivism* and *Universalism*. The first one is study by Anthropology and considers emotions as a social construction and not universal to all human as the view of *universalism* that is study by life science: psychology, neuroscience, biology, lately by computer science. *Social Constructivism* and *Universalism* begins a controversy about what standpoint has the truth with respect to emotion, this argument begins with Darwin's book *The Expression of the Emotions in Man and Animals*. Paul Ekman was a defensor of the *universalism* position, in his studies about face manifestations of emotions he considers that emotion does not depend of culture and it is manifested in the face.

Subsequent studies refute Ekman conclusions arguing that it is not possible to reproduce his studies and they have a high level of subjectivity interpreting Darwin's ideas to justify his position (Plamper, 2015).

Other important author is William James because he considers that emotions are tied in a strong way to the body, specifically to the brain (Plamper, 2015). In the same line of James appears Dane Carl Georg Lange considering that emotions are bounded to the body, to the brain in a special way, that is, the body receive the stimulus, that stimulus is a signal to the brain and the brain responds with another signal to the body, it is really important because this was before the exact knowledge of the nerves functions. With the focus of emotion study in the body the first psychological laboratory was founded in 1879 by Wilhelm Wundt, a psychologist that created his own theory of sense and considers emotions to be processed in the brain. Psychological laboratories give the introduction to a modern research, thinking in the possibility to recreate the experiments (crucial in modern research), but, the problem with respect to emotion was that it is not possible to capture the objectivity of it, for example, if scientist try to measure the increase of the blood pleasure trying to relate it to emotion study, how scientist know if an increased blood pleasure is related with emotion and not with some illness in a person? (Plamper, 2015).

In this way, the study of emotion through the body, through the brain takes another way due to an accident, the Phineas Gage accident in which a bar entered his head. This accident and others enable to study of the emotions via brain injuries. In the case of Phineas Gage that damaged his prefrontal cortex and scientist associated this damage to the alteration of his behaviour (more violent, easily angry) (Plamper, 2015) (see Figure 1.2).

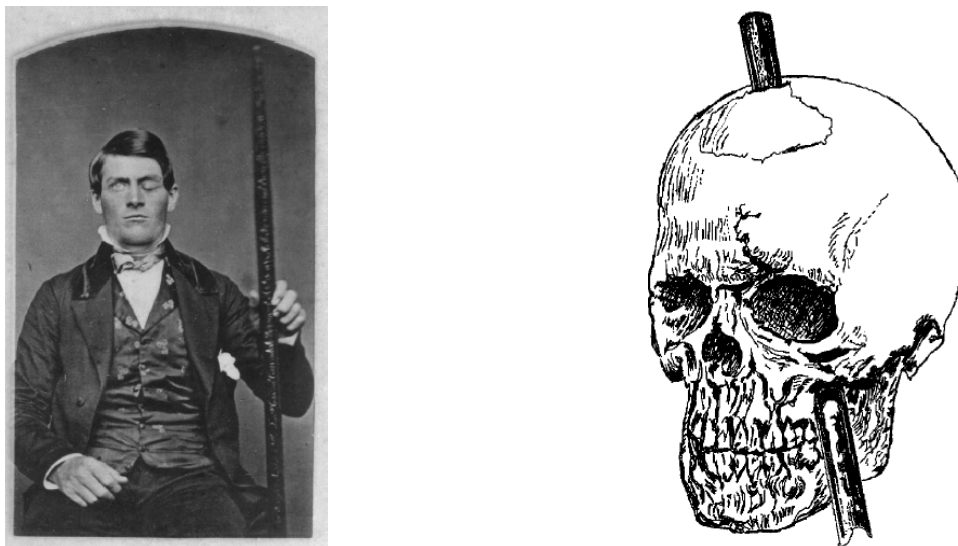


Figure 1.2: Phineas Gage and his accident

Posterior studies in beginnings of the twentieth century around the brain-emotions developed by Canon-Bar, Papez and MacLean are of huge importance (Plamper, 2015). Walter Canon, and later Philip Bar, found the thalamus region was with respect to emotions building a cir-

cuit between cortical part of the brain and subcortical regions of thalamus. The next important study by *Papez circuit* considers that the thalamus is linked to a cognitive path and to an emotional path, the cognitive path goes to the Cortex region's and it is composed of the cingulum, the hippocampus, the fornix and the mamillary bodies. The emotion path goes directly to the mamillary bodies, activating them and prompting the expression of emotions (Plamper, 2015). James Papez can be seen as the pioneer in neuroscience. Besides he also changes the view about the functionality of the brain considering brain activity as an interaction of the whole brain. The last important study developed by Paul D. MacLean considers that emotions produced in the brain are an interaction between three parts of the brain including the Papez Circuit and he added another regions. The union of the three regions is called the *Lymbic system* (see Figure 1.3).

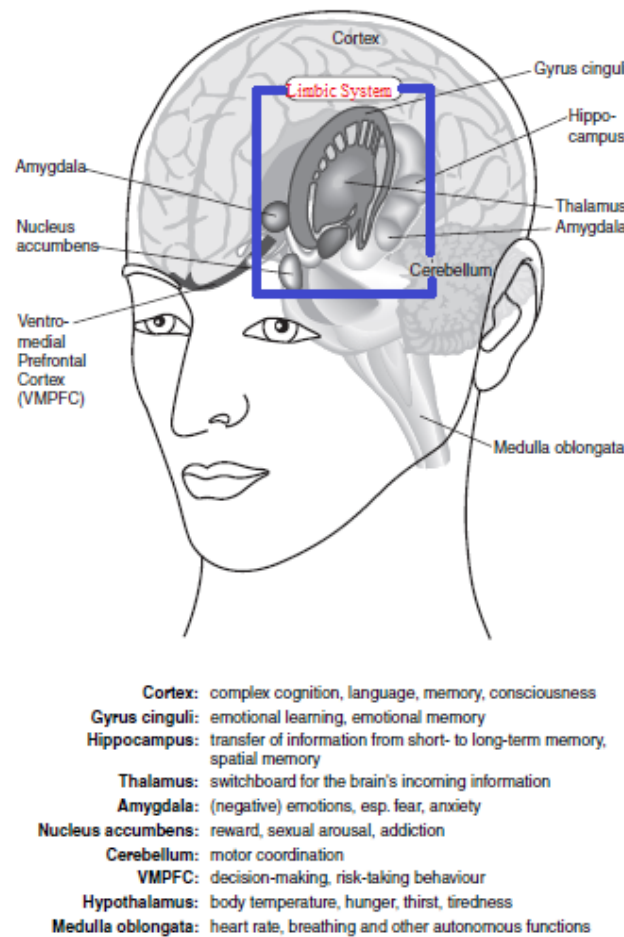


Figure 1.3: Limbic system as a result of Papez and MacLean studies

In 1950 the studies of emotion take a huge relevance in scientific studies. However, it is not clear why it takes so great importance. Part of the explanation (besides of science developments) is the relevance of philosophy of existentialism, rise of women's movement, influence of therapeutization, self-help groups, and New Age talk about one's own feelings (Plamper, 2015).

In this way, at the end of 1950, it comes the development of Appraisal theories with the hand

of Magda Arnold. The next chapter gives an extended overview of the appraisal theories of emotions.

Nowadays, the development of study of emotions increases in the applications of these studies, some will be mentioned in the following:

- History, as a social science, studies emotions to capture the emotional essence of some century, time, a social class, or try to compare societies from different times, for example, in Stearns and Stearns (1985), the authors create a new concept, *emotionology*, to capture in a better the emotions of a society at some period of human history. In (Stearns and Stearns, 1985), historians consider that an emotional approach to the study of history gives a better understand of the behaviour of the past people. For example, Stearns and Stearns (1985) is a study about the history of emotions developed by the historian express:

No longer can we assume without proof, as in a recent study of nineteenth-century grief, that people in the past shared our emotional experience, that we can use contemporary psychology to elucidate past behavior, or that we can use past data, without careful analysis, to bolster contemporary psychological theory. Linkages may be possible, but they require evaluation and evidence. They cannot be taken for granted as premises for research.

- Appraisal notion is used to help to characterize emotional disorders as *Anxiety, Depression, Guilt - Shame syndromes, Pride and disorders of the self, Anger disorders* (Scherer et al., 2001), using the notion of Appraisal that define emotions:

Some of the most successful psychological interventions now used to treat emotion disorders are guided by or are quite consistent with hypotheses in particular appraisal models (Scherer et al., 2001).

- Psychoanalysis is joined with neuroscience to redefine Freud's ideas around emotion and trauma (Plamper, 2015).

The aim of this research is to explore Appraisal theories of emotion, to study two theories that have a computational implementation. Those are *Scherer's theory: The Sequential Check Theory of Emotion Differentiation* (Scherer et al., 2001) and *The Cognitive Structure of Emotions* (Ortony et al., 1990). These theories have their own notion about emotion (it will be explained in next chapter), to give an introduction about emotion in Appraisal theories it is used the notion of emotion given by Magda Arnold and Richard Lazarus, first persons to formalize this concept in Appraisal theories:

Emotions are elicited and differentiated as a function of an individual's subjective interpretation or evaluation of important events or situations (Scherer et al., 2001)

In this way, *Emotion is the result of to Evaluate a situation presented to a subject*. This evaluation is defined as *Appraisal*. Arnold defines Appraisal as:

Appraisal is an “intuitive” assessment of the “here and now” aspects of situations and not a deliberative rational process (Scherer et al., 2001)

To give a better idea about emotion in Appraisal theories is showed in Figure 1.4. With respect to Figure 1.4, *Situation* is what enters the perception of a subject, it is then evaluated (appraisal) for the “machine” and then result the *emotion*. This is an intuition with respect to *emotion* and *appraisal* in appraisal theories. For that reason, in this work emotion it is written as appraisal-emotion to keep in mind that emotion is the result of an evaluated process in Appraisal theories.

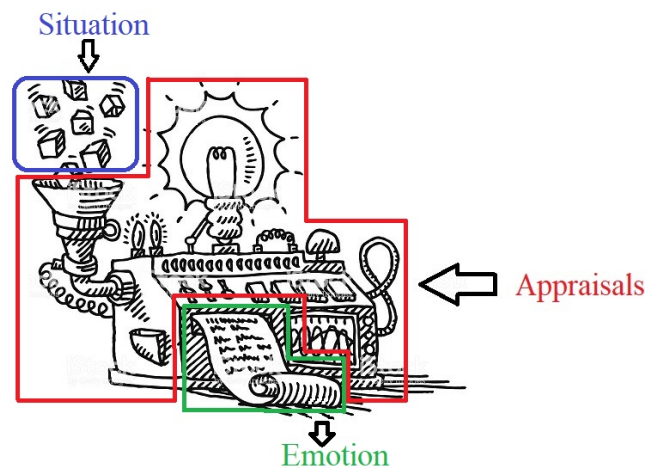


Figure 1.4: Intuition about Emotion in Appraisal Theories

In the computational field, there are also advances with respect to emotions. Those are manifested in the *Human Computer Interaction* (HCI) trying to make a more friendly this interaction or computational implementations are used to improves the theories itself (Marsella et al., 2010).

An important question in this area is: Why is it important to have computational models of emotions? There are two important reason: one reason is to understand the human emotion itself to help in emotional disorders (Scherer et al., 2001) and the second one reason is to improve the HCI (Marsella et al., 2010).

In this way, appraisal theories have a great number of computational implementations, especially two of them are of interest, OCC and Scherer’s theory. OCC is of interest maybe by its internal coherence that facilitates computational implementation (Adam et al., 2009) and Scherer’s theory is of interest because it has more than fifty years of study testing his theory with diverse experiments, comparing with neuroscience and making computational implementations (Scherer et al., 2001).

OCC was constructed in a way in which each emotion is perfectly differentiated from the others. For that reason it is possible to think of each emotion as a *category*¹. Category theory is a field of mathematics that establishes comparisons between objects. By this, it is possible to establish comparison between emotions (propose of this work) and possible between theories of emotions.

In some studies, Scherer (1993) retakes a semantic approximation to study emotion, studying the mechanism of emotion. The propose of this Thesis is to use its semantic approximation (Scherer, 1993) and to make a data analysis of it to provide a relationship between different emotions and so, provide the categories (in a mathematical senses) in Scherer's theory. For this the proposed model of Hartmann et al. (2012) is used, which is suggested the usage of categories.

In that way, the final propose of this thesis is to provide OCC and Scherer's theory of a categorical structure, and, in future works a comparison is possible which is a great importance for the main diversifies theories of emotions and its actual lack of comparison between them (Marsella et al., 2010).

1.1 Objectives

The research question of this work is: **How can category theory help to understand the study of emotions?**

In (Marsella et al., 2010) is explained the confusion that it is generated around the concepts *Appraisal* and *Emotion*. Appraisal in some theories is a cause of emotion, as in OCC, in others it is cause, part of the emotion experience, even part of the consequence of an emotion experience (Scherer et al., 2001).

The same happens with the concept of emotion, in some theories it is a valence reaction to types of situations, as in OCC (Ortony et al., 1990), in others it is a respond to an internal or external event involving process of information and types of behaviour as in Scherer's theory (Scherer et al., 2001). In OCC, there are 22 emotion types (discrete model of emotion) and in Scherer's theory there is a continuum space of emotions, in which each point of the space is an emotion (dimensional model of emotion). The quantity of concepts and its lack of precision in appraisal theories, the differences in theories with respect to structure does not allow an easy bridging between them. But as shown in (Marsella et al., 2010) it is necessary to establish this bridge to improve computational implementations of emotion theories and to improve emotion theories itself.

For these reasons, category theory allows to provide a mathematical relation of objects, where the objects can be seen as emotions. Hence, the general objective of this research is to propose categorical models of (part) of each theory using techniques of data analysis and category theory. It is mentioned "part" because the model is not proposed by the whole theory but by part of them.

¹category in a mathematical sense which it is explained in section 2.5.

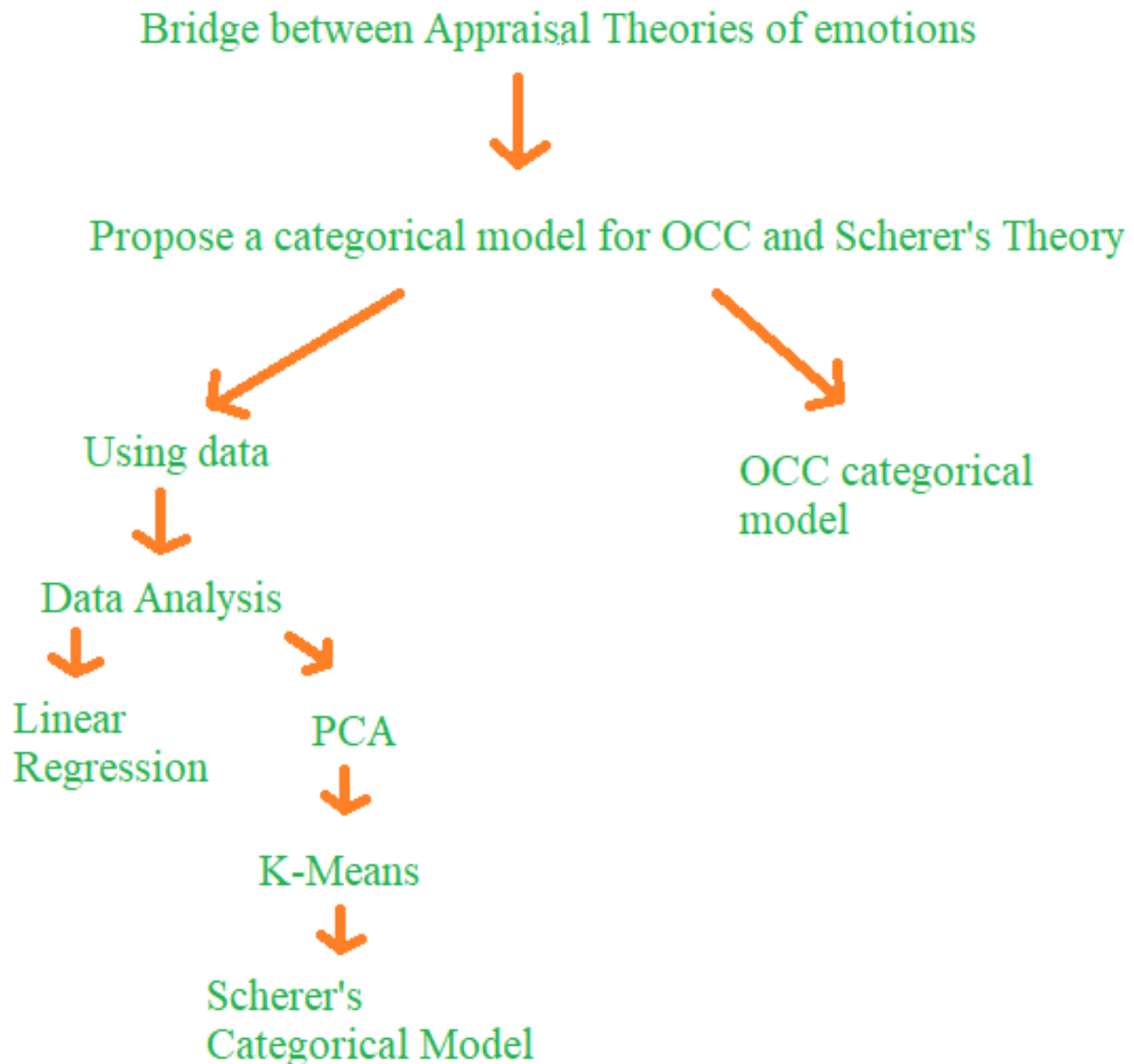


Figure 1.5: Map of the Research

Figure 1.5 shows each step of the research, it begins with the general propose of the research *To establish a bridge between Appraisal Theories of emotions*, it continues with a particular case developed in the research to achieve the general propose: *Propose a categorical model for OCC and Scherer's Theory*. After that, it follows with the develop of the categorical model for each theory, that is, for Scherer's theory is develop a data analysis based in his data from previews studies: **Using data, Data Analysis**. To realize the data analysis it is used different methods: *Linear Regression, PCA (Principal Component Analysis), K-Means*. Finally it is proposed the categorical model for Scherer theory.

With respect to OCC theory it is proposed directly the categorical model because OCC theory is a theory with a great internal coherence as it is showed through the research.

This general objective is divided in subobjectives:

- To study OCC and Scherer’s appraisal theories of emotion;
- To develop a data analysis for a data from a Scherer’s study
- To define a categorical models for each emotion theory;
- As a future work to propose a *categorical* relationship of those two emotion theories.

1.2 Struture of Work

This work is organized in the following way: section 2 gives an approach to Appraisal theories; section 3 exposes the empirical study develop by Scherer in 1993 to make a data analysis of it and establish the categories; section 4 shows the data analysis elaborated using Principal Component Analysis (PCA) and the data from 1993 Scherer’s study to provide the categories emotions; Section 2.5 expose the basic notions of category theory and develop the categories for Scherer and OCC theories; finally section 6 are the conclusions of this work as well as expose future works, those are a possible comparison of these theories using category theory that can be usefull for computational models.

2 APPRAISAL APPROACH TO EMOTIONS

Appraisal approach to emotions begins as a respond to the behaviorist approach to emotions, as, it responds to some particular questions and assume particular standpoints. The Appraisal theories of emotions try to respond to the following questions (Scherer et al., 2001):

- How can we account for the differentiated nature of emotional response?
- How can we explain individual and temporal differences in emotional response to the same event?
- How can we account for the range of situations that evoke the same emotion?
- What starts the process of emotional response?
- How can we explain the appropriateness of emotional responses to the situations in which they occur?
- What accounts for irrational aspects of emotions?
- How can developmentally and clinically induced changes in emotion be explained?

Also, appraisal theories of emotions adopt some assumptions that address these questions (Scherer et al., 2001):

- Emotions are differentiated by appraisals;
- Differences in appraisal can account for individual and temporal differences in emotional response;
- All situations to which the same appraisal pattern is assigned will evoke the same emotion;
- Appraisals precede and elicit emotions;
- The appraisal process makes it likely that emotions will be appropriate responses to the situations in which they occur;
- Conflicting, involuntary, or inappropriate appraisal may account for irrational aspects of emotions.

2.1 Some history of appraisal theories of emotions

Appraisal theories of emotions are based on the concept *Appraisal* developed by Magda Arnold in her *Cognitive Theory* in 1960 (Scherer et al., 2001). This theory was, according to (Scherer

et al., 2001) based on the hypothesis that the first step in emotion is an appraisal (evaluation) of the situation. The initial appraisals, according to Arnold, start the emotional sequence and arouse both the appropriate actions and the emotional experience itself, so that the physiological changes, recognized as important, accompany the actions and experiences. About *Appraisal* Magda says “*Appraisal* is an intuitive assessment of the ‘here and now’ aspects of situations and not a deliberative rational process” (Scherer et al., 2001).

In 1980, behaviorism considers emotions were not analysed with internal causes (Scherer et al., 2001). Thus, emotions are exposed in the behaviour, in facial expression of subjects like Charles le Brun in seventeenth century and Paul Eckman twentieth century like it was explained in the introduction of this work.

Richard Lazarus is other important researcher in the study of Appraisal Emotions and he based his model of emotions on Magda work (Scherer et al., 2001). Lazarus, in 1970, presents in a symposium his “cognitive perspective in emotion psychology”. He considers emotions in a continuous flux and postulated two principal appraisals:

1. *Primary appraisal*: directed at the establishment of the significance or meaning of the event to the organism (Lazarus et al., 1970);
2. *Secondary appraisal*: directed at the assessment of the ability of the organism to cope with the consequences of the event (Lazarus et al., 1970).

In this way, Magda proposes her ideas in 1960, Lazarus in 1970, but behaviorism does not pay attention to appraisal theories, even in 1980.

There was a path between behaviorism and appraisal theories. “*Two factor theory of emotions*” by Schachter is the theory that could be consider as the representative theory of emotions for more than 20 years (Scherer et al., 2001).

Schachter’s theory claims “that the quality of emotional experience is determined by a cognitive interpretation of situational cues brought about by the self-perception of an unusually high level of sympathetic arousal” (Scherer et al., 2001). Schachter continues the behaviorist line of investigation, he claims: “*individuals in a state of physiological arousal would show an emotional reaction and would describe the feelings they experienced as emotions*” (Scherer et al., 2001).

Thus, it is possible to think Schachter as the forerunner of Appraisals theories. Another forerunner of Appraisals theories were Phoebe Ellsworth, Ira Roseman and Craig Smith. Ira Roseman was a student in Yale and he works with Phoebe Ellsworth and Robert Abelson (Scherer et al., 2001). Ira Roseman presented in 1979 a manuscript about emotion scripts for a conference at the 87th Annual Convention of the American Psychological Association - APA (founded by Klaus Scherer).

Phoebe Ellsworth has a student in Stanford, Craig Smith, with him developed “some of the milestones of appraisal research and developed their own brand of appraisal theory” (Scherer et al., 2001). After he works with Phoebe Ellsworth he works with Richard Lazarus at Berkeley (Scherer et al., 2001).

Klaus Scherer and Nico Frijda are precursors of Appraisal theories. Klaus Scherer is a social psychologist. “He started develop a theory of emotion in order to provide a theoretical grounding for his empirical work on emotional expression. A preliminary version of the theory was published in 1979” (Scherer et al., 2001). He is looking for to differentiate emotion, so, he proposed in 1980 his “stimulus evaluation checks”. This work was the beginning of *component process model of emotion* “that Scherer continuously updates and submits to empirical testing” (Scherer et al., 2001). Other effort do it by Scherer is the need to do a comparison between appraisals of different models of emotions and to comment their convergence (Scherer et al., 2001).

Nico Frijda is other precursor of appraisal theories. He develop his theory based in the researchers mentioned above. In 1986 he published his book *The emotions* and in 1988 he published the book *The laws of emotions*, “for decades, nobody had dared to postulate laws in psychology” (Scherer et al., 2001). Frijda defines emotion as “states of action readiness” Scherer et al. (2001), that is, as “emotional states that underlie emotional behavior” (Scherer et al., 2001).

As a related work to the precursors it is possible to consider *The Cognitive Structure of Emotions* of Andrew Ortony, Gerald Clore and Allen Collins. They took the concept “cognitive appraisal” for their theory, this theory will be explain in detail later because it is a central subject in this job.

In 1980 comes a famous debate “Lazarus - Zajonc” or “cognition - emotion debate”. Zajonc thinks that affect and cognition are not interrelated in part because for him they have different neural systems. Lazarus thinks that cognition is necessary and sufficient as a causal antecedent of emotion. For Lazarus (1982) emotions is “a result of an anticipated, experienced or imagined outcome of an adaptationally relevant transition between organism and environment” (Scherer et al., 2001).

Scherer and Leventhal try to close the debate explaining that the debate was a semantic problem around if something is a cognitive process or not. Nico Frijda retakes the debate saying that it is necessary distinguish between cognition antecedents of emotions and cognitive structure of emotions. For Frijda emotional experience is not a linear problem with inputs and outputs, for him it is a recursive process (Scherer et al., 2001).

Lazarus-Zajonc debate was not have a definite conclusion, but it shows the importance of emotions in psychology research, shows the importance of cognition in the study of emotions and it gives pass to the study of appraisal theories of emotions (Scherer et al., 2001), beginning by Magda and continue by the precursors (in some cases, some of them continue until these days).

2.2 Scherer's and The Cognitive Structure of Emotion Appraisal theories of emotion

Appraisal theories are the most influential in computational modeling according to (Marsella et al., 2010). Figure 2.3 shows computational models based in these theories.

Scherer Appraisal Theory or The Sequential Check Theory of Emotion Differentiation

Scherers Appraisal theory is continuously evolving. He focus on testing his theoretical theory. Scherer describes emotions as a “theoretical construct that consists of five components corresponding to five distinctive functions” (Scherer et al., 2001). Table 1 shows the five components:

Function	Emotion Component	Organismic Subsystem
Evaluation of objects and events	Cognitive component	Information processing (CNS)
System regulation	Peripheral efference component	Support (CNS, NES, ANS)
Preparation and direction of action	Motivational component	Executive (CNS)
Communication of reaction and behavioral intention	Motor expression component	Action (SNS)
Monitoring of internal state and organism environment interaction	Subjective feeling component	Monitor (CNS)

Table 1: Relationship between the functions and components of emotion and the organismic subsystems that subserve them.

CNS: Central Nervous System, NES: Neuro-Endocrine System, ANS: Autonomic Nervous System, SNS: Somatic Nervous System. Above acronyms represent brain systems

These components are interrelated in a sequence of Appraisal that he named as *Stimulus Evaluation Checks* (SECs). This work is focus in the study of SECs.

As define Scherer, Appraisals are determined by the *Stimulus Evaluation Checks* (SECs).

In this way, the type of emotion and its intensity is determined by the “profile of results of the appraisal process based on these SECs” (Scherer et al., 2001). The SECs or principal appraisals are:

1. *Relevance*: “How relevant is this event for me? Does it directly affect me or my social reference group?” (Scherer et al., 2001)

This SEC is composed of the following features taken from the event:

- *Novelty Check*: is guided by the grade of *predictability* in the event, also by the grade of *familiarity* of the subject experience the emotion with the event. Also, *Suddenness* is the most primitive level of novelty, the lowest level of novelty detection (Scherer et al., 2001).
- *Intrinsic Pleasantness Check*: “The evaluation of whether a stimulus event is likely to result in pleasure or pain (in the widest sense)” (Scherer et al., 2001).
- *Goal Relevance Check*: “This check established the relevance, pertinence, or importance of a stimulus or situation for the momentary hierarchy of goals/needs” (Scherer et al., 2001).

2. *Implication Assessment*: If the consequences of the event put in danger or not the organism or its adaptation to the given environment.

This appraisal is composed of the following feature checks:

- *Causal Attribution Check*: To define the causes of an event, if it is an organism the cause of the events, its intentions.
- *Outcome probability Check*: The probability that certain consequences are materialized.
- *Discrepancy from Expectation Check*: “The situation created by the event can be consistent or discrepant with the individual’s expectation concerning that point in time or position in the action sequence leading up to a goal” (Scherer et al., 2001).
- *Goal/Need Conduciveness Check*: “The organism needs to check the conduciveness of a stimulus event to help attain one or several of the current goals/needs” (Scherer et al., 2001).
- *Urgency check*: “Adaptative action in response to an event is particularly urgent when high-priority goals/needs are endangered and the organism has to resort to fight or flight and/or when it is likely that delaying a response will make matters worse. Urgency is also evaluated on a continuous scale: the more important the goals/needs and the greater the time pressure, the more urgent immediate action becomes” (Scherer et al., 2001).

3. *Coping Potential Determination*: “The coping potential check determines which types of responses to an event are available to the organism and which consequences will affect the organism under each option. The net result of the evaluation on this check is the estimated degree of coping potential for the most promising response option available to the individual in this situation” (Scherer et al., 2001). This SEC is composed of the following features or checks:

- *Control Check*: “to what extent an event or its outcomes can be influenced or controlled by natural agents” (Scherer et al., 2001).
 - *Power Check*: “If control is possible, coping potential depends on the power of the organism to exert control or recruit others to help” (Scherer et al., 2001).
 - *Adjustment Check*: “Organisms can adjust, adapt to, or live with the consequences of an event more or less well after all possible means of intervention have been used” (Scherer et al., 2001).
4. *Normative significance Evaluation*: This appraisal takes into account the evaluation do it by the person experiences the emotion about other persons, what other people think about the actions performed by the person experience the emotion (Scherer et al., 2001). This appraisal is composed of two features or checks:
- *Internal Standards Checks*: “This check evaluates the extent to which an action falls short of or exceeds internal standards such as one’s personal self-ideal (desirable attributes) or internalized moral code (obligatory conduct).” (Scherer et al., 2001).
 - *External Standards Checks*: “Social organization in groups implies shared values and rules (norms) concerning status hierarchies, prerogatives, desirable outcomes, and acceptable and unacceptable behaviors” (Scherer et al., 2001). This check is about external norms, that is, what others think about the consequence of an action.

The process of appraisal in Scherer’s Theory is a unique sequential process, where relevance is the first appraisal in the sequence. Also, it left the possibility of a parallel process of appraisals, as in artificial intelligence or in the process of babies where their firsts appraisals, with *Relevance* and *Implications Assessment* (Scherer et al., 2001).

In Scherer’s theory, appraisals are observed in a sequence process that are in accordance with *Levels of Processing in Appraisal: sensory-motor, schematic and conceptual level*. In this way, appraisals first occur in the lowest level: sensory-motor detecting the event with “natural reflex and innate feature detections” (Scherer et al., 2001). The schematic level is composed of information learning history of the individual “which can be conceptualized as abstract representations of learned responses to specific stimulus patterns” (Scherer et al., 2001). On the conceptual level information is undertaken as criteria of evaluation, as a reflexive process (Scherer et al., 2001). Conceptual level is related with the capacity to reappraise the event in a conscious way, because also, it is possible to appraise in an unconscious way (Scherer et al., 2001), as the way in which we response with a fear emotion.

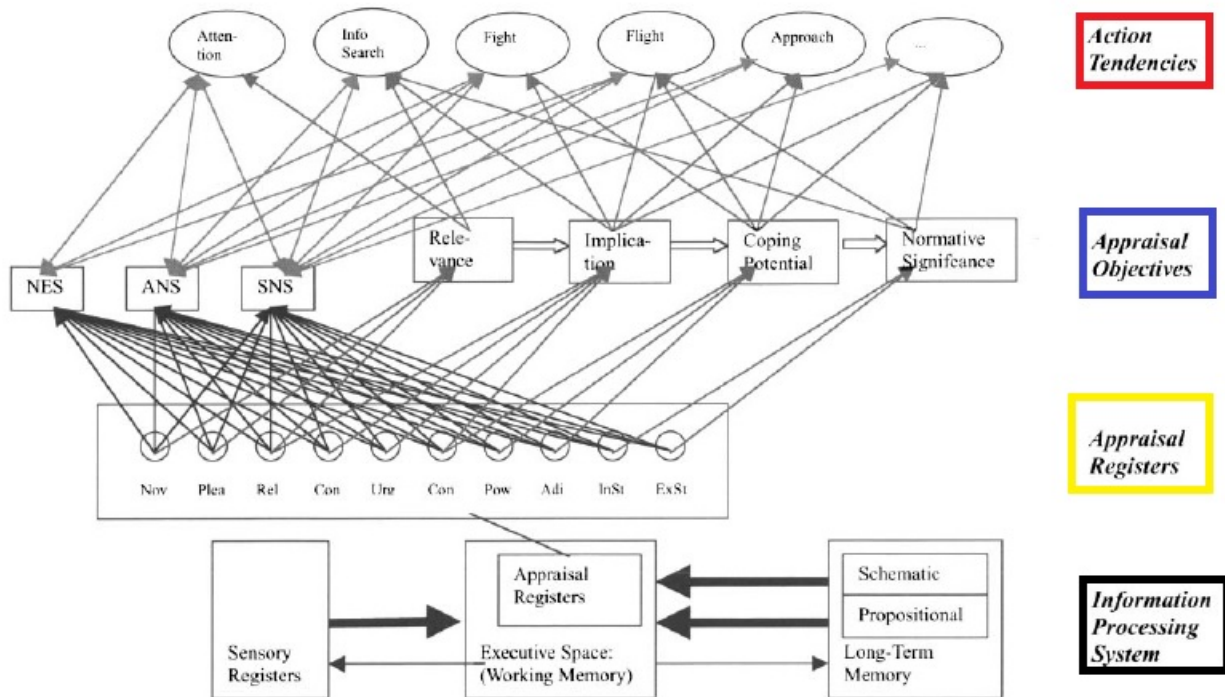


Figure 5.3. Sketch of the potential architecture of the appraisal process as part of a general information processing system, separately driving peripheral support systems and alternative action tendencies.

Figure 2.1: Structure of appraisals in Scherer's theory, adapted from (Scherer et al., 2001).

Figure 2.1 shows the structure of appraisal process. First of all is the appraisal register of information (yellow color and explained in paragraphs before figure 2.1), after that comes the evaluation process in the sequence of appraisals showed in four appraisals mentioned (blue color) (Scherer et al., 2001). Acronyms *NES*, *ANS*, *SNS* were explain in table 1. These systems are affected by the appraisal and help to generate *Actions Tendencies* (red color), these actions can be of behaviour, changes in breathing, physical changes. In this work, it is explore just *Appraisal Objectives* (blue colore) and *Appraisal Registers* (yellow color), it is because to establish future comparison between Appraisal theories, *Appraisal notions* are the basic concepts shared by all appraisal theories, for example, OCC does not study brain systems implications in its theory, so, it is not possible to compare this part with Scherer's theory which it explore it.

2.3 Cognitive Structure of Emotions: The OCC Theory

OCC theory is an appraisal theory of emotions, in which each emotion is discrete, totally differentiated one from another. Here, emotions are “*valenced reactions to events, agents, or objects with their particular nature being determined by the way in which the eliciting situation is constructed*” (Ortony et al., 1990). In this theory, a subject perceives his world in three ways Events, Agents and Objects according to (Ortony et al., 1990):

1. Events refers to its consequences, that is, a subject construct a situation as an answer

to the consequences of the event.

2. Agents refers to its actions, that is, a subject construct a situation as a respond to agent actions
3. Objects refers to its properties like object (physical, abstract), that means: a subject construct a situation in accordance with the properties perceive of the object, for example: color, smell, taste, ...

A reaction to each of the situations constructed by the subject is associated to different emotions, it is to say, there are specific emotions related to the perception of the world by: *Events*, *Actions*, *Agents*, for example: Happy-for corresponds as reaction to *Events* and Love is perceive as a reaction to *Objects* (can be: a person, ideas, material objects, animals) as shows figure 2.2. The intensity of these emotions are determined by the *eliciting conditions*, they are variables taken from the situation constructed by the subject. Eliciting conditions will active or not some respective emotion (Ortony et al., 1990).

Emotions are discrete and are organized in sets called *Emotion types*, for example (continue with example before): Happy-for and Love are emotion types. Emotions types are sets of similar emotions, each emotion types keeps all the words that are similar to Happy-for or Love (continuing with the example).

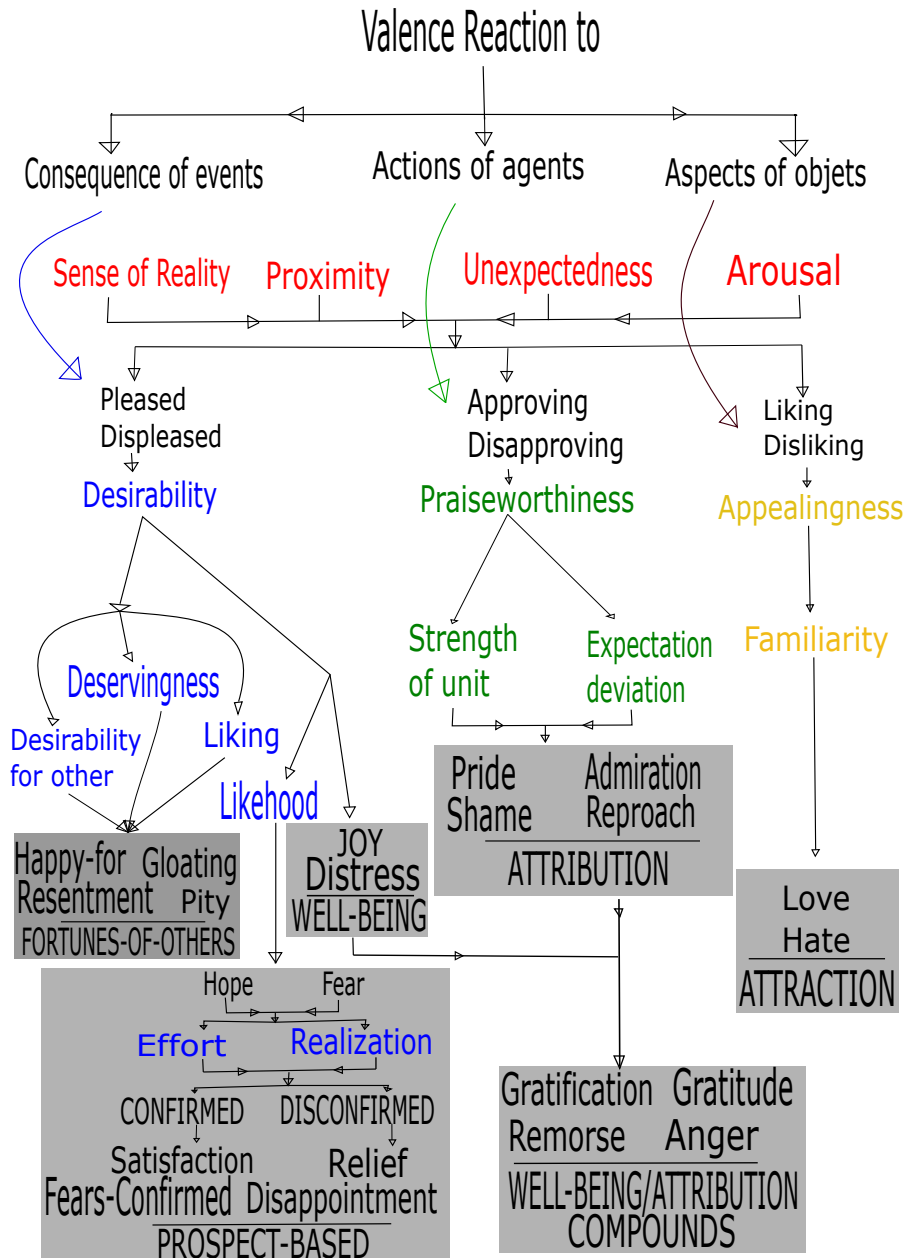


Figure 2.2: Structure of Appraisals in OCC, adapted from (Ortony et al., 1990)

Figure 2.2 shows in red color global variables that affect all the type of emotions; in blue color the variables that affect just type of emotions elicited by *reactions to events*; in green color the variables that affect just type of emotions that are *reactions to agent actions*; and finally in yellow color the variables that affect just type of emotions that are *reactions objects or aspect of them*. Boxes of gray color represents sets of emotion types. An *Emotion type* is a set of words that represents similar emotions that can be group in a representative word, for example, type of emotion *Joy* represents the set of words: contented, cheerful, delighted, ecstasies, elated, euphoric, feeling good, glad, happy, joyful, jubilant, pleasantly surprised, pleased, etc (Ortony et al., 1990), it will happen for each type of emotion in OCC. Also, OCC proposes set of type of emotions which in Figure 2.2 are grouped by boxes of gray color. Emotion types grouped in a same box means that are elicited by the same variables

but with difference intensity.

This organization does not mean that there are basic emotions that form the others emotions in which some few emotions are mixtured to form the whole quantity of emotions. This organization means that there is a successive differentiation in the sense that the eliciting conditions of certain emotions always appear before or at the same time that the eliciting conditions of other emotion, for example, the eliciting conditions of reproach appears before that the eliciting conditions of Anger (Ortony et al., 1990).

To describe the cause of human emotions it is necessary to describe the appraisal structure, this structure can be described thinking in the action of a person. The action of a person requires motivation and this motivation needs *goals*, *standards* (*beliefs*) and attitudes (interests).

Goals refers to objectives that have a subject. *Standards* are the state of affairs that one believes *ought* to obtain (Ortony et al., 1990). *Standards* are beliefs and moral judgemental evaluation. *Attitudes* are *dispositional* liking (or disliking) one has for certain objects, or attributes of objects (Ortony et al., 1990).

Int this way, *Events* are linked with *goals*, *Agents* with *Standards* and *Objects* with *Attitudes*. So, *Goals*, *Standards* and *Attitudes* are linked to the cause of emotions, this linked is give it by the *Eliciting Condition* (determine the intensity of an emotion) described as *Global Variables* and *Local Variables*. *Global variables* affect the intensity of all emotions. These variables are *Sense of reality* (*SR*), *Proximity* (*PR*) and *Unexpectedness* (*UN*) (Ortony et al., 1990) (in red color in figure 2.2).

SR explains situations as: emotions in a dream or emotions seeing a movie or emotions reading a book. When a person has emotions in this case, it is because of *sense of reality* is high. This variable affects the intensity of all emotions and if its values is low then the intensity of emotions is low also (Ortony et al., 1990).

PR measures the proximity of emotions, that is, if a subject constructs a situation that arises an emotion several times and he creates again this situation in a short time then the intensity emotion is high because the same emotion arises in a short time. OCC consider that there are empirical registers that justified the existence of this global variable and there are concrete registers about fear emotion (Ortony et al., 1990), for example, in persons with anxiety in which fear is continually arised and the *proximity* in time of the arise increase the intensity of the emotion.

PR has a relation with *SR*: if *SR* is low, that is, if the situation has an low level sense of reality, then *PR* has no much influence in the intensity of emotions, but if *SR* has a high level sense of reality, then if *PR* also has a high level of proximity then *PR* and it has a high influence in the intensity of emotions.

UN evaluate the degree of surprise of some Event, Action Agent of Aspect of an Object, if this variable is *high* so the intensity of the emotion experience is high, the same happens if this variable is *low* (Ortony et al., 1990).

The last global variable is *Arousal* (*Ar*), this is a physiological variable but it is a variable

that affect cognitive process and for this, that affect emotions. *Ar* measures the alteration of the person before the situation. Increases in these variables intensify the experienced emotion (Ortony et al., 1990).

In addition to this *Global Variables*, there are *Local Variables* that measure specific emotions groups. Emotions are divided in three big groups, each one determined by the perception of the world: *Events*, *Agents Actions*, *Objects*, inside of each big group there are *Local Variables* that affect each particular group of emotion. *Desirability (D)*, *Praiseworthiness (P)* and *Appealingness (Ap)* are *Local Variables* (blue, green and yellow color respectively in figure 2.2), *D* affects just all the emotions elicited by *Events*, *P* affects all the emotions elicited by *Agent Actions* and *Ap* affect all the emotions elicited by *Objects*.

Other local variable are *Likelihood (L)* which is a local variable that measures the intensity of emotions as *hope*, *fear*, *relief*, these emotions have part of the Prospect Based Emotions affected just by *Events*. *L* has to do with anticipated events and it is forward-looking. *L* is different from *UN* because *UN* is assessed after an event and it backward-looking (Ortony et al., 1990). An example to show the difference between these variables: a person runs every day for a park during one month, the last day he meets with an old friend and he is *surprised*. Here, it is the *UN* variable that measures the intensity of the emotion. An example for *L* is the following: the same person running for the park during one month. In this case, he observes a car that come in his direction, he is surprised and thinks that it is not probably that the car run over him. In this case the emotions is measure by *L*.

Other local variable that affects emotions about events is *Effort (E)*. This variable is related with the implementation of plans. That is, the possibility of increase *D* (desirability) of a goal can increase as a result of *E* (effort) expended in the implementation of plans to achieve it. Sometimes, *E* is related with *L*. For example: a person can increase *L* following a plan and making effort *E* to do it. Other local variable is *Realization (Re)* affected by *Events*, that takes into account the intensity of positive emotions related to *goals*.

Other local variable is *Desirability-for-other (Dfo)*, that concerns with the set of *Type of emotions* (remember: a type of emotions is a set of emotions): Fortunes-of-others. *Dfo* affects the intensity of the emotion felt by the person reacting to another's fortunes or misfortunes. The degree of this variable is evaluating the other person with his own cognitive structure and suppose an state in the other person.

Liking (Lk) is other local variable, that measures the degree of like or dislike that feel the person that experience the emotion for the other, it is a measure of affect that feel a person for another.

Another local variable affected by *Events* is *Deservingness (Ds)*, that tends to capture if some event was fair for a person or not, if the person experiencing the emotion consider that it is fair, then the intensity of desirability for positive emotions increase (Ortony et al., 1990). Now, local variables that affect just emotions affected by *Actions of agents* are: *Strength of cognitive unit (SCU)*, that stablishes the possibility of to have a bond with someone else or something else in the sense that can cause emotions like *Pride* or *shame*. When SCU is high,

the person experiencing the emotion (formal agent) has a strong bond with himself or with other person or group of persons. For example, when a person has shame of the politicians of his country, that happen because the person has a strong bond with his country (Ortony et al., 1990). Other local variable is *Expectation-deviation (ED)*, that is a type of *Unexpectedness* (The global variable) but only influence the set of *type of emotion: Attribution emotions*. Its unexpectedness refers to a role of person that occupied in a society, for example: a shoemaker that save the life of a person in the sea, this is unexpectedness in the role that occupied a shoemaker, then the intensity of the emotion associated to the person that observe the action of a shoemaker is more high than the intensity of the emotion that emerge in the case of a lifeguard because its role is expected (Ortony et al., 1990).

Finally, local variables affected by *Objects* are: *Familiarity (Fa)* which expresses that if a person is related with an object for a period of time it is possible that the intensity of attraction emotions change (Ortony et al., 1990).

These are the variables that elicit the emotions in OCC. Now, with respect to the emotions, as it was explain earlier, there are *type of emotions* that are set of emotions, each type of emotions represents similar emotions affected by the global and local variables, like shows figure 2.2. The type of emotions are organized in sets as shows figure 2.2 (in purple color):

1. *Fortunes of others* is composed of the set of type of emotions: *Happy-for*, *Gloating*, *Resentment*, *Pity*.
2. *Prospect Based* is composed of the set of type of emotions: *Hope*, *Fear*, *Fears confirmed*, *Relief* and *Disappointment*.
3. *Well-Being* is composed of the type of emotions: *Joy* and *Distress*.
4. *Attribution* is composed of the set of type of emotions: *Pride*, *Admiration*, *Shame* and *Reproach*.
5. *Compound of Well-Being/Attribution* is composed of the type of emotions: *Gratification*, *Gratitude*, *Remorse* and *Anger*.
6. *Attraction* is composed of the set of type of emotions: *Love* and *Hate*.

So, *emotions* are elements of a *type of emotions* and *type of emotions* are grouped in these six sets (exposed above). *Global* and *Local variables* affects emotions depending on the perception of the world by: *Events*, *Actions of agents* and *Objects*

2.3.1 Relations between Groups of type of emotions in OCC

There is a relationship between the groups of type of emotions WELL-BEING and PROSPECT-BASED emotions in the following way: PROSPECT-BASED emotions has two subsets of type of emotions, one subset is *Confirmation Emotions* and the other is *Disconfirmation Emotions*, like shows the figure 2.2.

Confirmation Emotions is the set of type of emotions represented by the words *Satisfaction* and *Fears-Confirmed* and disconfirmation emotions are represented by the words *Dissapointment* and *Relief*. It is possible that when one emotion of *Satisfaction* or of *Fears-Confirmed* emerges is followed by one emotion of the group WELL-BEING EMOTIONS, as *Joy* or *Distress* (Ortony et al., 1990). For example, if emerges an emotions of satisfaction, it is possible that emerges an emotion of joy, and if emerges an emotion of fears-confirmed, it is possible that emerges an emotion of *Distress*.

About *Compound Emotions*, this set is a result of a mixture of type of emotions: WELL-BEING and ATTRIBUTION type of emotions, as shows figure 2.2.

2.4 Computational Models of Emotion

Computational models of emotion begin as a response to different fields of investigation (Marsella et al., 2010), as *Human Computer Interaction*, *Enhance general models of intelligence*, *Understanding human behaviour*. These models are based in emotions theories showed with its respective computational models in Figure 2.3. *Appraisal theories* consider *Appraisals* to be the cause of emotions. Most of the computational models are developed based on these theories, by its internal coherence (Adam et al., 2009) or the simplicity of the appraisal model of emotions (Scherer et al., 2001) or by its emphasis on emotions as computable artifacts (Lin et al., 2012).

Advantage of computational models are to improve, to question emotions theories, to reduce conceptual confusion between theories (Marsella et al., 2010). For example, using EMA, a computational model based on appraisal theories and showed in Figure 2.3:

EMA was used to generate specific predictions about how human subjects will apraise and cope with emotional situations and argue that empirical tests of these predictions have implications for psychological appraisal theory (Marsella et al., 2010).

Computational models have more than fifty years of research (Marsella et al., 2010).

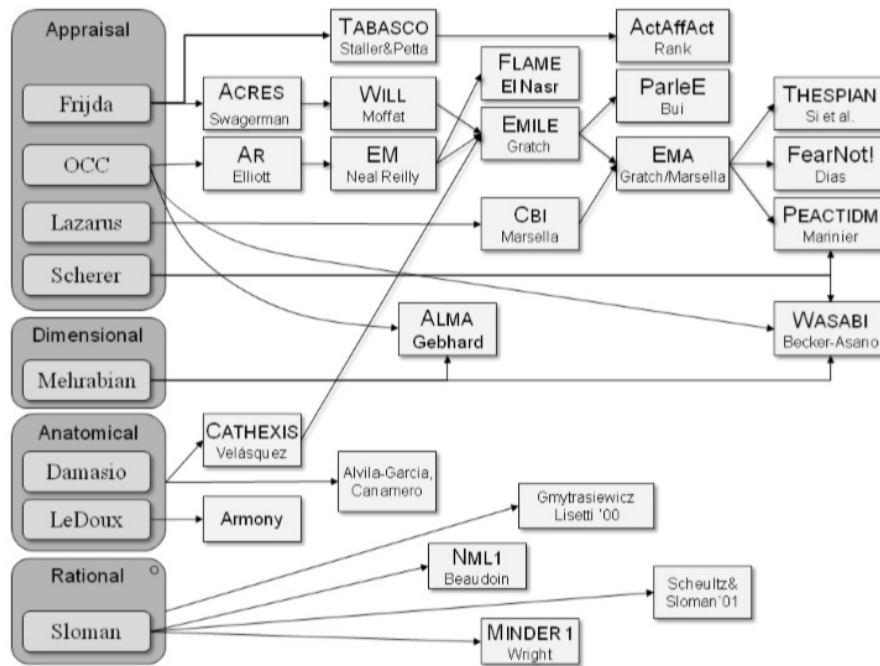


Figure 2.3: Computational models of emotions (Marsella et al., 2010)

Appraisal computational models are based in the assumption that emotion follows an evaluation process of a situation that elicit the emotion (Marsella et al., 2010).

Dimensional computational models observe emotions in a dimensional space in which each emotion is a point in the space (Marsella et al., 2010).

Anatomical computational models try to reconstruct the neural links that underlie organism emotional reactions and based on this to think emotions as neural circuits (Marsella et al., 2010).

Rational computational models conceive emotions as adaptational functions to the environment, and each emotion is an adaptative behaviour that it is implemented in a model intelligence (Marsella et al., 2010).

According to Lin et al. (2012), *Dimensional* and *Appraisal* theories of emotion are similar in a way that both map emotion-evoking events to emotional states. The difference is that appraisal theories maps discrete events in discrete emotions and dimensional theories maps discrete events in a continuous of orthogonal dimensions.

It is important to note how computational models of emotions share emotion theories, for example, ALMA and FLAME adopt OCC appraisal variables and EMA model are inspired in Lazarus theory (Marsella et al., 2010).

2.5 Category Theory

Category Theory (CT) was introduced by Samuel Eilenberg and Saunders Mac Lane in 1945 and according to Krömer (2007): "During the 1950s and 1960s, CT became an important conceptual framework in many areas of mathematical research, especially in algebraic topol-

ogy and algebraic geometry is consider a functional language in mathematics”.

CT began in 1960s, but its applications are recent, for example: Massachusetts Institute of Technology (MIT) began in January of 2019 an online course (Spivak and Fong, 2019) about *Applied Category Theory* given by professors Brendan Fong and David Spivak who wrote the book (Fong and Spivak, 2019), explaining some of the possibilities to applied CT in computer science, robotics, even explains how develops of professor David Spivak were the base for a programming language based in CT called Categorical Query Language (CQL) (develop by Ryan Wisnesky) generalizing concepts of Structured Query Language (SQL) as it is explain in the website of CQL (Wisnesky, 2015).

In this way, this section introduces the basics notions of CT (following definitions from (Mac Lane, 2013)) to a next step implement this CT notions in OCC and Scherer’s theories.

To define a *Category*, it is necessary to define a *Graph* because the notion of Category is a constructive notion and the first notion in the construction is a graph. A Graph is a set O of objects, a set A of arrows, and two functions *Domain* (dom) and *Codomain* (cod):

$$Domain := (dom : A \rightarrow O) \quad (2.1)$$

which assigns to each arrow $f \in A$ an object $a \in O \Rightarrow$ It is define $a := dom f$

$$Codomain := (cod : A \rightarrow O) \quad (2.2)$$

which assigns to each arrow $f \in A$ an object $b \in O \Rightarrow$ It is define $b := cod f$.

Or in a simple notation for both definitions: $a \xrightarrow{f} b$. This notation was used in Figure 2.4.

In this graph, the *set of composable pairs of arrows* is the set:

$$Ax_O A = \{\langle f, g \rangle \mid g, f \in A \wedge dom g = cod f\} \quad (2.3)$$

called the “*Product over O*”.

The above set is the set of all possible composition of arrows, that is, if $\langle f, g \rangle \in Ax_O A$ then exist $g \circ f$ which it is the *composition arrow of f and g* as shows Figure 2.4.

Next step is to define a *category*. A *category* is a *graph* with two additional functions and two additional axioms hold:

Functions:

1. *Identity function* which assigns to each object c an arrow id_c :

$$O \xrightarrow{id} A \quad (2.4)$$

$$c \rightarrow id_c \quad (2.5)$$

where $id_c = 1_c : c \rightarrow c$ is an arrow from an object c to itself, that is: $dom(id_c) = c = cod(id_c)$, id_c is called *identity arrow*.

2. *Composition function* which assigns to each pair of arrows $\langle g, f \rangle$ an arrow $g \circ f$, that is:

$$A \times_O A \xrightarrow{\circ} A \quad (2.6)$$

$$\langle g, f \rangle \longrightarrow g \circ f \quad (2.7)$$

Where $dom(g \circ f) = dom f$, $cod(g \circ f) = cod g$. Figure 2.4 illustrates the idea of the *composition function*:

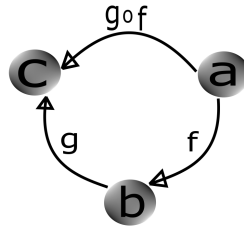


Figure 2.4: Illustration of composition function

It is necessary to define *Axioms* to complete the definition of a *category*.

Axioms:

1. *Associativity*. For given objects and arrows in the configuration:

$$a \xrightarrow{f} b \xrightarrow{g} c \xrightarrow{h} d$$

it always has the equality:

$$h \circ (g \circ f) = (h \circ g) \circ f \quad (2.8)$$

2. *Unit law*. For all arrows $f : a \rightarrow b$ and $g : b \rightarrow c$ composition with the identity arrow 1_b gives:

$$1_b \circ f = f \quad \text{and} \quad g \circ 1_b = g \quad (2.9)$$

Next step is to define the notion of *Functor* to establish the relationship between Categories.

Functor:

According to Mac Lane (2013):

A *functor* is a morphism of categories. In details, for categories C and B a functor $T : C \rightarrow B$ with domain C and codomain B consists of two suitably related functions: The *object function* T , which assigns to each object c of C an object Tc of B and the *arrow function* (also written T) which assigns to each arrow $f : c \rightarrow c'$ of C an arrow $Tf : Tc \rightarrow Tc'$ of B , in such a way that:

$$T(1_c) = 1_{Tc}, \quad T(g \circ f) = Tg \circ Tf \quad (2.10)$$

The latter whenever the composite $g \circ f$ is defined in C . A functor, like a Category, can be described in the “arrow-only” fashion: it is a function T from arrows f of C to arrows Tf of B , carrying each identity of C to an identity of B and each composable pair of $\langle g, f \rangle$ in C to a composable pair $\langle Tg, Tf \rangle$ in B , with $Tg \circ Tf = T(g \circ f)$.

Figure 2.5 shows the intuition of a Category and a Functor. A *Category* exhibits the relation between objects in it through arrows (functions, relations, ...) that preserve properties (composition between objects, associativity between arrows, ...). In Figure 2.5 arrows are exhibit with letters: f, g . A Functor preserves relation between categories sending an image of the category objects and an image of the arrows. In Figure 2.5 is showed through letter F .

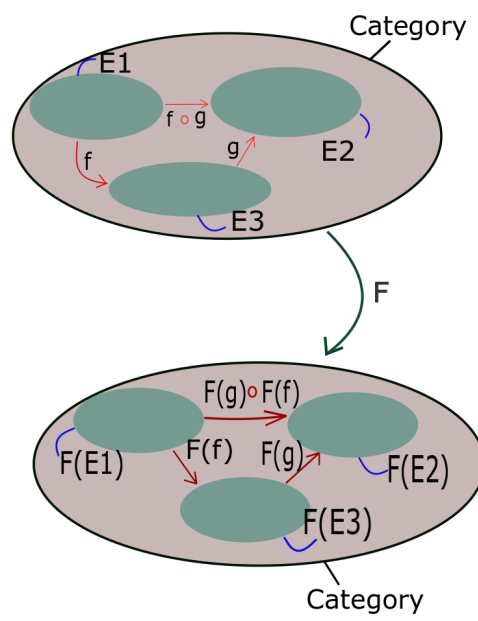


Figure 2.5: Intuition of a *Category* and a *Functor*

3 SCHERER'S EMPIRICAL STUDY TESTING HIS THEORY

Scherer and colleagues developed some studies to support his theory: *The Sequential Check Theory of Emotion Differentiation*. One of them is (Scherer, 1993), here, it is developed a testing in which 256 persons participated. The idea of the study is to predict the emotions experienced by people. They have to remember a situation in which they experienced an emotion and this situation is reconstructed responding 15 questions, each one (or a few) is related to one SECs (see section 2.2 to remember what are SECs) . The order of the questions is presented in a way that follow the sequence process of appraisals (SECs) in Scherer's theory Scherer (1993). Figures 3.1 and 3.2 show the questions:

1. Did the situation that elicited your emotion happen very suddenly or abruptly?
[SEC1-NOVELTY]
(0) not pertinent (1) not at all (2) a little (3) moderately (4) strongly (5) extremely
 2. Did the situation concern an event or an action that had happened in the past, that had just happened or that was to be expected for the future? [see text]
(0) not pertinent (1) the event had happened a long time ago
(2) it happened in the recent past (3) it had just happened at that moment
(4) it was to be expected for the near future (5) it was to be expected in the long run
 3. This type of event, independent of your personal evaluation, would it be generally considered as pleasant or unpleasant? [SEC2-INTRINSIC PLEASANTNESS]
(0) not pertinent (1) very unpleasant (2) rather unpleasant (3) indifferent
(4) rather pleasant (5) very pleasant
 4. Was the event relevant for your general well-being, for urgent needs you felt, or for specific goals or plans you were pursuing at the time? [SEC3-RELEVANCE]
(0) not pertinent (1) not at all (2) a little (3) moderately (4) strongly (5) extremely
 5. Did you expect the event and its consequences before the situation actually happened?
[SEC3-EXPECTATION]
(0) not pertinent (1) never in my life (2) not really (3) I did not exclude it
(4) a little (5) strongly
 6. Did the event help or hinder you in satisfying your needs, in pursuing your plans or in attaining your goals? [SEC3-CONDUCTIVENESS]
(0) not pertinent (1) it hindered a lot (2) it hindered a little (3) it had no effect
(4) it helped a little (5) it helped a lot
 7. Did you feel that action on your part was urgently required to cope with the event and its consequences? [SEC3-URGENCY]
(0) not pertinent (1) not at all (2) a little (3) moderately (4) strongly (5) extremely
-
- (Continued)

Figure 3.1: Questions associated to SECs (Scherer, 1993)

8. Was the event caused by your own actions—in other words, were you partially or fully responsible for what happened? [SEC4–CAUSATION]
- (0) not pertinent (1) not at all (2) a little, but unintentionally
 (3) somewhat, but I was unaware of the consequences
 (4) quite responsible, I knew what I was doing
 (5) fully responsible, I absolutely wanted to do what I did
9. Was the event caused by one or several other persons—in other words, were other people partially or fully responsible for what happened? [SEC4–CAUSATION]
- (0) not pertinent (1) not at all (2) a little, but unintentionally
 (3) somewhat, but he/she/they were unaware of the consequences
 (4) quite responsible, he/she/they knew what they were doing
 (5) fully responsible, he/she/they absolutely wanted to do what they did
10. Was the event mainly due to chance? [SEC4–CAUSATION]
- (0) not pertinent (1) not at all (2) a little, but human action was the decisive factor
 (3) somewhat, but human action contributed to it (4) strongly (5) exclusively
11. Can the occurrence and the consequences of this type of event generally be controlled or modified by human action? [SEC4–CONTROL]
- (0) not pertinent (1) not at all (2) a little (3) moderately (4) strongly (5) extremely
12. Did you feel that you had enough power to cope with the event—i.e. being able to influence what was happening or to modify the consequences? [SEC4–POWER]
- (0) not pertinent (1) not at all (2) a little (3) moderately (4) strongly (5) extremely
13. Did you feel that, after having used all your means of intervention, you could live with the situation and adapt to the consequences? [SEC4–ADJUSTMENT]
- (0) not pertinent (1) not at all (2) with much difficulty (3) somewhat (4) quite easily
 (5) without any problem at all
14. Would the large majority of people consider what happened to be quite in accordance with social norms and morally acceptable? [SEC5–NORM COMPATIBILITY]
- (0) not pertinent (1) certainly not (2) not really (3) probably (4) most likely
 (5) certainly
15. If you were personally responsible for what happened, did your action correspond to your self-image? [SEC5–SELF COMPATIBILITY]
- (0) not pertinent (1 was not responsible) (1) not at all (2) not really (3) somewhat
 (4) strongly (5) extremely well
-

Figure 3.2: Questions associated to SECs (Scherer, 1993)

Each answer of a question has a numerical value associated, at the end, for each situation given by a person, it is obtained a vector represented by each numerical value associated to each answer of the question.

In this study, the total quantity of emotions to have into account is 14, and for each emotion it is associated a vector and its values were taken from previous studies (Scherer, 1988). In this way, the prediction is performed associating the vector given by a person to some of the 14 vectors represented emotions. In that way, the emotion associated as predicted is the emotion that has the smaller euclidean distance to some of the 14 vectors, as shows figure 3.3, which the point V represents the emotion associated to the person with the questionnaire and the 14 points represents the vector emotions taken from previous Scherer study (Scherer, 1988). In this example, the point (from the 14 points) with smaller distance to the point V is the point 8 with distance d_1 , and it is associated the vector emotion 8 to the person.

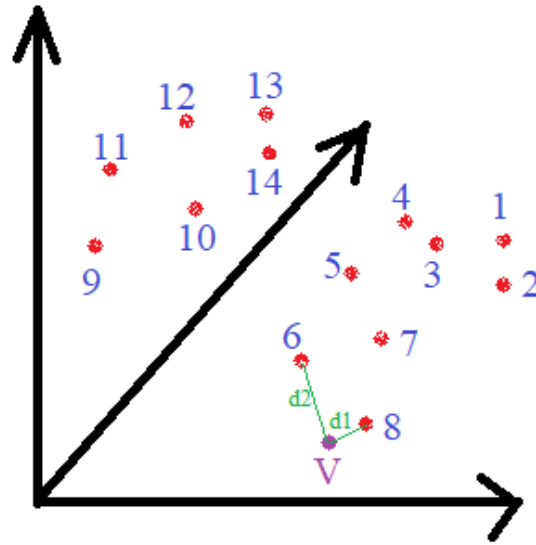


Figure 3.3: Example of emotion prediction in Scherer study

The values obtained of the 256 people are presented in Figures 3.4, 3.5 and 3.6. It is necessary to clarify that next results are a mean (in a mathematical sense) of the 256 predictions, that is, subjects with the same predicted emotion are taken a *mean* of the vector associated to them through the questions and this *mean* represent the vector emotion of the subjects that later it is compare with the vector of prediction developed by Scherer in (Scherer, 1988). For example, 3 persons have the same predicted emotion of *Joy*, and it is taken the vector of each one product of the 15 questions, then, it is taken a *mean* of each component of the vectors to obtain a final vector representing these 3 persons with the same emotion associated of *Joy*.

Emotion	Nov	Time	Plea	Role	Expec	Condu	Urgen	EgoC	OhnC	ChaC	Cont	Power	Adjus	Ext	Int
Enjoy	2.59	2.19	4.41	2.70	3.59	2.96	1.52	3.11	3.41	2.19	3.22	2.37	3.33	3.85	2.93
	2.00	2.00	4.00	3.00	4.00	4.00	1.00	2.00	3.00	2.00	0.00	0.00	5.00	0.00	0.00
	-0.59	-0.19	-0.41	0.30	0.41	1.04	-0.52	-1.11	-0.41	-0.19	*	*	1.67	*	*
	1.18	0.86	0.70	1.57	0.88	0.87	0.95	1.38	1.29	1.06	1.55	1.20	1.21	1.09	1.44
Joy	3.29	2.97	4.26	3.76	2.85	3.97	2.62	3.12	2.97	2.71	3.32	2.59	3.53	3.97	2.88
	4.00	3.00	3.00	4.00	1.00	5.00	2.00	3.00	3.00	3.00	0.00	0.00	4.00	0.00	0.00
	0.71	0.03	-1.26	0.24	-1.85	1.03	-0.62	-0.12	0.03	0.29	*	*	0.47	*	*
	1.30	0.93	0.99	1.30	1.16	0.98	1.39	1.24	1.44	1.24	1.40	1.26	1.14	0.93	1.56
Disgus	3.10	2.30	1.70	1.60	1.90	2.40	2.40	2.00	2.70	3.30	2.30	1.70	2.20	2.70	1.50
	3.00	3.00	1.00	2.00	2.00	3.00	3.00	1.00	3.00	3.00	3.00	2.00	4.00	3.00	0.00
	-0.10	0.70	-0.70	0.40	0.10	0.60	0.60	-1.00	0.30	-0.30	0.70	0.30	1.80	0.30	*
	1.68	1.44	0.98	0.92	0.90	1.20	1.60	1.20	1.30	1.10	1.36	0.84	1.36	1.56	1.60
Contem	2.33	1.00	1.67	1.67	2.33	1.00	2.67	2.00	4.00	1.00	2.33	1.33	3.67	3.00	1.00
	2.00	2.00	2.00	2.00	2.00	3.00	2.00	1.00	5.00	1.00	3.00	4.00	4.00	2.00	0.00
	-0.33	1.00	0.33	0.33	-0.33	2.00	-0.67	-1.00	1.00	0.00	0.67	2.67	0.33	-1.00	*
	1.11	1.33	0.89	0.89	1.11	0.67	1.11	0.67	0.67	0.00	1.78	0.89	0.89	0.67	1.33
Sad	3.06	2.21	1.85	2.44	2.65	2.38	2.26	2.15	2.62	2.21	3.12	1.71	2.26	2.26	1.56
	2.00	2.00	2.00	4.00	2.00	2.00	2.00	2.00	3.00	3.00	3.00	0.00	2.00	0.00	0.00
	-1.06	-0.21	0.15	1.56	-0.65	-0.38	-0.26	-0.15	0.38	0.79	-0.12	*	-0.26	*	*
	1.24	1.15	0.97	1.58	1.39	0.94	1.27	1.07	1.46	1.25	1.59	0.91	0.87	1.17	1.12
Desper	4.12	2.74	1.79	2.90	2.17	2.00	3.34	1.95	2.53	2.38	2.52	1.50	1.69	1.90	1.47
	4.00	3.00	2.00	5.00	1.00	1.00	3.00	1.00	2.00	3.00	3.00	0.00	1.00	0.00	0.00
	-0.12	0.26	0.21	2.10	-1.17	-1.00	-0.34	-0.95	-0.53	0.62	0.48	*	-0.69	*	*
	0.88	0.91	1.05	1.98	1.07	0.93	1.51	0.98	1.42	1.53	1.41	0.72	0.54	1.27	1.27

Figure 3.4: Vector (Row) of: **subject emotion**, **emotion predicted**, **difference of the first two vectors** and standard deviation of empirical values given by the subject in the questions (Scherer, 1993)

Emotion	Nov	Time	Plea	Rele	Expec	Condu	Urgen	EgoC	ObtC	ChaC	Cont	Power	Adju	Ext	Int
Anxi	2.42	2.74	1.26	3.05	3.16	3.00	2.79	2.32	2.11	2.16	2.95	2.42	2.68	3.05	1.68
	2.00	5.00	2.00	3.00	2.00	2.00	3.00	1.00	4.00	3.00	3.00	2.00	3.00	0.00	0.00
	-0.42	2.26	0.74	-0.05	-1.16	-1.00	0.21	-1.32	1.89	0.84	0.05	-0.42	0.32	•	•
	1.43	1.10	0.49	1.42	1.11	0.74	1.51	1.28	1.39	1.25	1.64	1.07	0.58	1.41	1.35
Fear	4.32	2.79	1.42	2.79	2.05	2.11	3.53	2.11	2.11	2.37	3.42	2.26	1.47	1.00	1.58
	5.00	4.00	2.00	5.00	1.00	1.00	5.00	1.00	4.00	4.00	2.00	1.00	2.00	0.00	0.00
	0.68	1.21	0.58	2.21	-1.05	-1.11	1.47	-1.11	1.89	1.63	-1.42	-1.26	0.53	•	•
	0.94	1.05	0.71	1.80	0.82	0.88	1.44	0.99	1.20	1.48	1.30	1.36	0.97	0.84	1.51
Irrit	3.27	2.64	2.18	2.73	3.45	2.36	3.09	2.55	3.36	1.55	3.55	2.45	2.91	3.00	2.27
	3.00	2.00	3.00	3.00	2.00	2.00	3.00	2.00	4.00	3.00	4.00	3.00	4.00	2.00	2.00
	-0.27	-0.64	0.82	0.27	-1.45	-0.36	-0.09	-0.55	0.64	1.45	0.45	0.55	1.09	-1.00	-0.27
	1.21	1.19	1.02	1.70	0.96	0.76	1.02	0.96	0.76	0.69	1.21	1.14	0.84	0.91	1.16
Anger	2.63	2.05	1.32	2.79	2.11	2.21	3.26	1.53	2.89	1.63	4.21	2.68	2.42	1.74	2.11
	4.00	3.00	2.00	4.00	1.00	1.00	4.00	1.00	4.00	2.00	4.00	4.00	3.00	1.00	0.00
	1.37	0.95	0.68	1.21	-1.11	-1.21	0.74	-0.53	1.11	0.37	-0.21	1.32	0.58	-0.74	•
	1.37	1.10	0.60	1.51	1.08	1.13	1.09	0.93	1.39	1.15	1.00	1.17	0.88	1.01	1.29
Indiff	3.67	3.67	2.33	2.67	3.67	2.33	1.67	0.67	2.67	1.67	3.67	1.67	3.67	2.33	2.33
	1.00	3.00	3.00	2.00	5.00	3.00	1.00	0.00	0.00	0.00	0.00	3.00	4.00	0.00	0.00
	-2.67	-0.67	0.67	-0.67	1.33	0.67	-0.67	•	•	•	-0.67	1.33	0.33	•	•
	0.44	0.44	0.89	1.56	1.78	0.44	0.89	0.44	1.56	0.44	0.44	0.44	1.78	1.56	1.11
Shame	4.50	2.75	1.50	2.00	1.50	2.50	1.25	4.00	2.75	1.50	3.50	2.50	2.50	3.00	3.75
	2.00	3.00	2.00	4.00	2.00	2.00	2.00	4.00	2.00	1.00	2.00	2.00	3.00	2.00	2.00
	-2.50	0.25	0.50	2.00	0.50	-0.50	0.75	0.00	-0.75	-0.50	-1.50	-0.50	0.50	-1.00	-1.75
	0.50	0.38	0.50	2.00	0.50	1.50	0.38	1.00	1.25	0.75	1.50	1.50	1.25	1.00	0.75

(Continued)

Figure 3.5: Vector (Row) of: **subject emotion**, **emotion predicted**, **difference of the first two vectors** and standard deviation of empirical values given by the subject in the questions (Scherer, 1993)

Emotion	Nov	Time	Plea	Rele	Expec	Condu	Urgen	EgoC	OthC	ChaC	Cont	Power	Adjus	Ext	Int
Guilt	3.33	3.33	1.67	0.33	1.33	1.67	3.33	3.33	2.33	1.00	3.00	2.00	1.67	3.67	2.33
	2.00	2.00	3.00	4.00	2.00	3.00	3.00	4.00	1.00	1.00	2.00	2.00	3.00	2.00	1.00
	-1.33	-1.33	1.33	3.67	0.67	1.33	-0.33	.067	-1.33	0.00	-1.00	0.00	1.33	-1.67	-1.33
	1.56	0.44	0.44	0.44	0.44	1.11	1.56	0.44	1.78	0.00	2.00	1.33	1.11	1.11	1.11
Pride	1.11	1.67	4.44	3.67	3.67	2.78	2.00	3.44	1.89	1.33	4.00	2.89	3.11	3.56	3.33
	2.00	3.00	4.00	4.00	4.00	4.00	2.00	5.00	2.00	2.00	4.00	4.00	5.00	4.00	5.00
	0.89	1.33	-0.44	0.33	0.33	1.22	0.00	1.56	0.11	0.67	0.00	1.11	1.89	0.44	1.67
	0.62	1.26	0.99	1.56	1.19	2.02	1.33	1.85	1.85	0.81	1.11	1.70	1.65	1.38	1.56

Note: First row: Mean empirical input vector. Second row: Theoretical predicted vector. Third row: Difference between empirical and theoretical vectors. Fourth row: Standard deviation for values in empirical vector. * signifies cases in which difference score is not meaningful because prediction is "0—not pertinent or open".

Abbreviations: Nov, novelty; Time, when did the event happen; Plea, intrinsic pleasantness; Rele, relevance; Expec, expectation; Condu, goal conduciveness; Urgen, urgency; EgoC, self as causal agent; OthC, other(s) as causal agent; ChaC, chance as causal agent; Cont, control; Power, Power to cope; Adjus, capacity to adjust; Ext, external; Int, internal.

Figure 3.6: Vector (Row) of: subject emotion, emotion predicted, difference of the first two vectors and standard deviation of empirical values given by the subject in the questions (Scherer, 1993)

It is possible that the vector of prediction with the smaller distance to the vector of the subject not be the emotion experienced by the person. In this case, it is assigned the vector emotion with the next smaller distance to the vector subject, and, again if the vector emotion does not correspond to the emotion experienced the person choose the emotion between the 14 emotions and it is assigned to the vector given by the person. If the emotion experience

by the subject is out of the 14 emotions, the person label the emotion experienced and it is assigned the vector subject to the label given by him/her (Scherer, 1993). It is possible that there are emotions misses in prediction and also emotions with dubious prediction, each set of emotions goes to the categories of *Total Misses* and *Dubious Misses*.

The final results of this study is presented in Figure 3.7. In this figure, where each columns represents:

- *Total Situat*: represents the total of situations presented in each emotion;
- *1st Hit*: represents the total of situations that match in the first try of prediction;
- *2nd Hit*: Represents the situation that match in the second try;
- *Total Misses*: represents the total of situations that were not possible to associate an emotion in the first two *Hits*;
- *Dubious Misses*: there are cases that consider as isolated points and it is not possible of a classification with certainty;
- *% Correct*: represents the percentage of total hits (first plus second) minus the *Dubious Misses*.

It is possible to observe that the emotions *Anxiety/Worry*, *Fear/Terror* have a good number a situations, but they do not have a good prediction. This study takes into account this result to improve the method, as takes into account others possible appraisals and the notion of time: past, present or future of the situation (Scherer, 1993). Emotions as *Indifference/Boredom*, *Embarrassment/shame*, *Guilt feelings* have been a few of situation, and the study can not conclude about its precision. Emotions as *Anxiety/Worry*, *Fear/Terror* have a low quantity of match, maybe because the complexity of the emotions and it is not possible to capture its estructure in the study (Scherer, 1993).

<i>Emotion</i>	<i>Total Situat</i>	<i>1st Hit</i>	<i>2nd Hit</i>	<i>Total Misses</i>	<i>Dubious Misses</i>	<i>% Correct</i>
Happiness/Feel good	27	21	1	5	2	88.0
Joy/Elation	34	30	3	1	1	100.0
Displeasure/Disgust	10	4	3	3	1	77.8
Contempt/Scorn	3	3	0	0	0	100.0
Sadness/Depression	34	19	3	12	3	71.0
Desperation/Grief	58	49	8	1	1	100.0
Anxiety/Worry	19	2	0	17	5	14.3
Fear/Terror	19	2	2	15	4	26.7
Irritation/Cold anger	11	3	4	4	1	70.0
Hot anger/Rage	19	10	4	5	1	77.7
Indifference/Boredom	3	1	0	2	0	33.3
Embarrassment/Shame	4	1	0	3	1	33.3
Guilt feelings	3	1	0	2	1	50.0
Pride/Jubilation	9	6	0	3	1	75.0
Totals	253	152	28	73	22	77.9

Figure 3.7: Final results of Scherer (1993)

4 DATA ANALYSIS OF SCHERER'S STUDY FROM 1993 USING (PCA) PRINCIPAL COMPONENT ANALYSIS

Principal Component Analysis (PCA) is a mathematical algorithm that study the variation of a data set reducing the dimensionality of the data (Ringnér, 2008). In this work, PCA is used because it enables to reduce dimensionality of Scherer's data, to reduce dimensionality allows to obtain a representation in a plane of two dimension, also to use PCA because it is a first step to establish a cluster of Scherer's data, that is to establish a cluster of the emotions. PCA has a great variety of implementation in different fields from mathematical education (Alves et al., 2015) (Machado and Pereira, 2012) to generic studies.

PCA accomplishes the process of decrease dimensionality finding directions, called *principal components*, in which the variation of the data is maximal in these directions (Ringnér, 2008). Data in the context of PCA are organized as vectors in a vector space $(\mathbb{R}^n, n \in \mathbb{N})$. In this way, the *principal components* are new vectors that are organized according to how much of the variation present in the data they contain (Ringnér, 2008). In this way, the first *principal component* contains the greater variation of the data, the second *principal component* contains the second greater variation of the information, and so on.

It is important to stand out that PCA is designed to identify directions with the largest variations of information in data and not directions to identify clusters of data. Ringnér (2008) gives an example in which data of gens has two classes of receptors, but PCA does not separated these two classes of data. Also, Ringnér (2008) recommends that PCA is used as a first step before a clustering of information.

4.1 PCA algorithm

It is necessary to define some mathematical objects to begin with the algorithm. Let $\mathbf{x}$ be a vector of p components (variables), these components are taken as random.

According to Jolliffe (2002):

The first step is to look for a linear function $\alpha_1'x$ of the elements of $\mathbf{x}$ having maximum variance, where α_1 is a vector of p constants $\alpha_{11}, \alpha_{12}, \dots, \alpha_{1p}$, and $'$ denotes transpose, so that

$$\alpha_1'x = \alpha_{11}x_1 + \alpha_{12}x_2 + \dots + \alpha_{1p}x_p = \sum_{j=1}^p \alpha_{1j}x_j \quad (4.1)$$

Same happens with α_2 with the property that α_2 is uncorrelated to α_1 in the sense that are perpendicular, that is, its inner product is zero: $\alpha_2' \cdot \alpha_1 = 0$. The idea is to look for α_i until a k th α_k that is uncorrelated to the above $\alpha_1, \alpha_2, \dots, \alpha_{k-1}$. The k th $\alpha_k'x$ is the k th Principal

Component. The maximum of Principal Components is p (the number of the components of the vectors $\mathbf{x}$ and α_i) (Jolliffe, 2002).

According to Jolliffe (2002):

Having defined Principal Components, we need to know how to find them. Consider, for the moment, the case where the vector of random variables $\mathbf{x}$ has a known covariance matrix Σ . This is the matrix whose (i,j) th element is the (known) covariance between the i th and j th elements of $\mathbf{x}$ when $i \neq j$, and the variance of j th element of $\mathbf{x}$ when $i = j$. The more realistic case, where Σ is unknown, follows by replacing Σ by a sample covariance matrix $\mathbf{S}$. It turns out that for $k = 1, 2, \dots, p$, the k th Principal Component is given by $z_k = \alpha'_k x$ where α_k is an eigenvector of Σ corresponding to its k th largest eigenvalue λ_k . Furthermore, if α_k is chosen to have unit length ($\alpha' \alpha = 1$), then $\text{var}(z_k) = \lambda_k$, where $\text{var}(z_k)$ denotes the variance of z_k .

To find the form of the Principal Component it is necessary to maximize the variance of the eigenvectors α_i in the following way: $\text{var}[\alpha'_1 x] = \alpha'_1 \Sigma \alpha_1 = \lambda$. After, to maximize the variance of the eigenvectors is to find the eigenvalue with maximum value. For that reason, for α_1 correspond the eigenvalue with the maximum value λ_1 , for α_2 correspond the eigenvalue with the second largest eigenvalue λ_2 , and so on, for α_k correspond the k th largest eigenvalue (Jolliffe, 2002).

4.2 Correlation Analysis using SPSS

In this work, PCA was used in the data of the study described in chapter 3. To this propose is used the software SPSS² to analysis the results.

First of all, it is showed the emotion vectors taken from the study (Scherer, 1993), in Figures 4.1 and 4.2.

²Statistical Analysis Software: www.ibm.com/SPSS/demo.

Pride	Guilt	Enjoy	Joy	Disgust	Contempt	Desperation
2,00	2,00	2,00	4,00	3,00	2,00	4,00
3,00	2,00	2,00	3,00	3,00	2,00	3,00
4,00	3,00	4,00	3,00	1,00	2,00	2,00
4,00	4,00	3,00	4,00	2,00	2,00	5,00
4,00	2,00	4,00	1,00	2,00	2,00	1,00
4,00	3,00	4,00	5,00	3,00	3,00	1,00
2,00	3,00	1,00	2,00	3,00	2,00	3,00
5,00	4,00	2,00	3,00	1,00	1,00	1,00
2,00	1,00	3,00	3,00	3,00	5,00	2,00
2,00	1,00	2,00	3,00	3,00	1,00	3,00
4,00	2,00	,00	,00	3,00	3,00	3,00
4,00	2,00	,00	,00	2,00	4,00	,00
5,00	3,00	5,00	4,00	4,00	4,00	1,00
4,00	2,00	,00	,00	3,00	2,00	,00
5,00	1,00	,00	,00	,00	,00	,00

Figure 4.1: Emotion vectors taken from the study (Scherer, 1993)

Irritation	Shame	Indifference	Anger	Fear	Anxiety	Sad
3,00	2,00	1,00	4,00	5,00	2,00	2,00
2,00	3,00	3,00	3,00	4,00	5,00	2,00
3,00	2,00	3,00	2,00	2,00	2,00	2,00
3,00	4,00	2,00	4,00	5,00	3,00	4,00
2,00	2,00	5,00	1,00	1,00	2,00	2,00
2,00	2,00	3,00	1,00	1,00	2,00	2,00
3,00	2,00	1,00	4,00	5,00	3,00	2,00
2,00	4,00	,00	1,00	1,00	1,00	2,00
4,00	2,00	,00	4,00	4,00	4,00	3,00
3,00	1,00	,00	2,00	4,00	3,00	3,00
4,00	2,00	3,00	4,00	2,00	3,00	3,00
3,00	2,00	3,00	4,00	1,00	2,00	,00
4,00	3,00	4,00	3,00	2,00	3,00	2,00
2,00	2,00	,00	1,00	,00	,00	,00
2,00	2,00	,00	,00	,00	,00	,00

Figure 4.2: Emotion vectors taken from the study (Scherer, 1993)

As showed the chapter 3, there are 14 emotions and the values in Figures 4.1 and 4.2 represent the expected values that each check has to have to elicit the emotion, according to (Scherer, 1993).

After the emotion vectors, it is showed the correlation matrix, in which it is possible to see some correlation between emotions (Figure 4.3).

	Enjoy	Joy	Disgust	Contempt	Pride	Guilt	Desperation	Irritation	Shame	Indifference	Anger	Fear	Anxiety	Sad
Enjoy	1,000	,749	,209	,274	,068	,362	,142	,131	,225	,485	-,060	,162	,326	,479
Joy	,749	1,000	,315	,140	-,222	,440	,470	,107	,322	,066	,152	,510	,412	,584
Disgust	,209	,315	1,000	,577	-,459	-,069	,282	,454	-,166	,193	,501	,410	,553	,314
Contempt	,274	,140	,577	1,000	-,148	-,038	-,060	,643	-,045	,350	,582	,110	,466	,159
Pride	,068	-,222	-,459	-,148	1,000	,392	-,557	-,263	,468	,310	-,516	-,766	-,521	-,351
Guilt	,362	,440	-,069	-,038	,392	1,000	,207	-,094	,747	,246	,067	,066	-,036	,273
Desperation	,142	,470	,282	-,060	-,557	,207	1,000	,349	,190	-,031	,624	,895	,600	,798
Irritation	,131	,107	,454	,643	-,263	-,094	,349	1,000	-,113	,088	,734	,419	,480	,460
Shame	,225	,322	-,166	-,045	,468	,747	,190	-,113	1,000	,087	,080	,079	,087	,251
Indifference	,485	,066	,193	,350	,310	,246	-,031	,088	,087	1,000	,118	-,139	,305	,105
Anger	-,060	,152	,501	,582	-,516	,067	,624	,734	,080	,118	1,000	,724	,668	,444
Fear	,162	,510	,410	,110	-,766	,066	,895	,419	,079	-,139	,724	1,000	,709	,681
Anxiety	,326	,412	,553	,466	-,521	-,036	,600	,480	,087	,305	,668	,709	1,000	,654
Sad	,479	,584	,314	,159	-,351	,273	,798	,460	,251	,105	,444	,681	,654	1,000

Figure 4.3: Correlation matrix obtained from software SPSS

Each value in the correlation matrix represents the $\cos(\theta)$, where θ is the angle between the vectors (emotions) (Machado and Pereira, 2012). For example, that explain why the matrix is symmetric (*symmetric* is a concept from linear algebra) and the principal diagonal has value 1 in all the components of it.

After the correlation matrix, it is established the *principal components vectors*, in this case was obtained three vectors, but software SPSS does not show these vectors. It shows the emotion components in the space formed by principal component vectors and their coordinates (Figure 4.4).

	Componente			
	1	2	3	4
Enjoy	,401	,629	,262	-,542
Joy	,596	,525	-,167	-,419
Disgust	,641	-,214	,369	-,164
Contempt	,486	-,113	,763	,125
Pride	-,617	,599	,314	,269
Guilt	,141	,836	-,091	,305
Desperation	,797	,046	-,497	,110
Irritation	,650	-,252	,371	,305
Shame	,106	,773	-,159	,474
Indifference	,164	,395	,647	-,091
Anger	,802	-,250	,124	,476
Fear	,872	-,129	-,420	,023
Anxiety	,856	-,047	,119	-,048
Sad	,802	,265	-,220	-,058

Figure 4.4: Emotions vectors in the space of principal components

In this study was produced a rotation of the space of principal components. This rotation is called *Normalized Varimax Rotation*, and it consists in rotate each principal component vector in a constant angle, with the propose to maximize the variation in data. Then, software SPSS shows the components of each emotion vector in the new space of principal components

rotated (Figure 4.6).

	Componente			
	1	2	3	4
Enjoy	,083	,112	,163	,934
Joy	,511	-,041	,196	,730
Disgust	,300	,610	-,283	,280
Contempt	-,067	,897	-,066	,184
Pride	-,734	-,126	,595	,047
Guilt	,085	-,049	,859	,268
Desperation	,927	,091	,165	,047
Irritation	,317	,780	-,031	-,080
Shame	,109	-,050	,916	,082
Indifference	-,306	,466	,210	,506
Anger	,589	,735	,098	-,224
Fear	,955	,194	-,031	,053
Anxiety	,606	,548	-,054	,286
Sad	,737	,202	,216	,367

Figure 4.5: Vector emotions rotated by the Varimax normalized method

Componente	1	2	3	4
1	,787	,544	,027	,289
2	-,102	-,191	,795	,566
3	-,608	,745	-,084	,260
4	,016	,334	,600	-,727

Figure 4.6: Principal component vectors rotated by Normalized Varimax method

The last step expressed in the software SPSS is to show the graph of the emotion vectors in the space rotated (Figure 4.7).

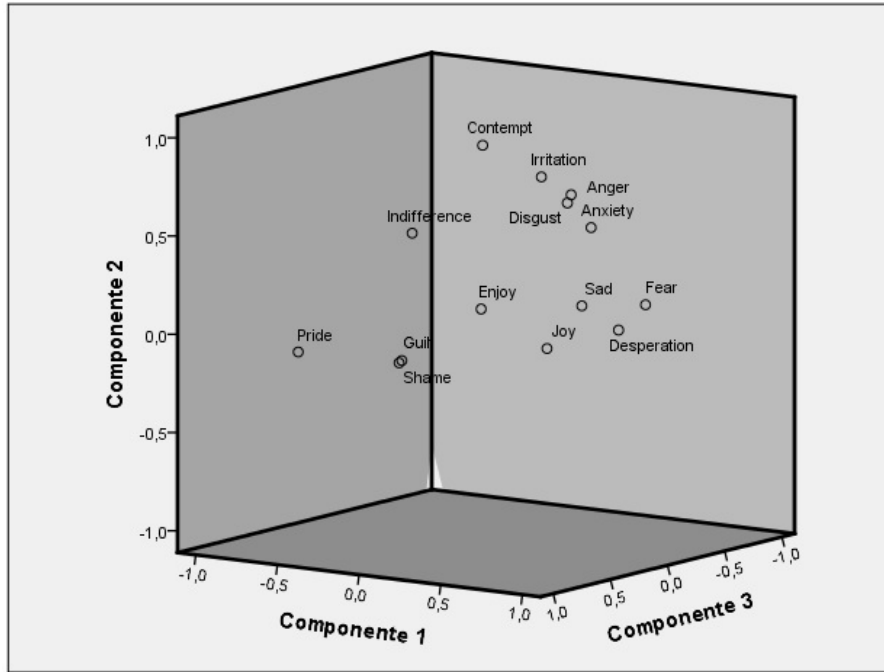


Figure 4.7: Graph of emotion vectors in the space of principal components rotated

Unfortunately, Figure 4.7 does not show a good perspective to show tendency of variance in emotion vectors. Also, software SPSS has a limited quantity of functions with respect to PCA. For these reasons it is used the software R to study Scherer's data with the propose to get results more clear.

4.3 Pearson Correlation and PCA analysis using R

Software R offers difference ways to study data, besides of this, it is well known that R is a free software improved by the contributions around the world. In this way, it is study the vector emotions using Correlation Pearson Coefficient as a first approximation to Scherer's data. In this first approximation it is used a scatter plot to compare the vectors correspond- ing to the emotions as show Figure 4.8. This scatter plot expose a Correlation Coefficient Pearson (r) to show the correlation between vector emotions. It is used Correlation Coeffi- cient Pearson because it explains if the vector emotions has a linear relationship (Hall, 2015), that is, if a coefficient is close to 1 ($r \approx 1$) it means that two emotions has similar values in each component, it can be interpreted as: *it is possible to classify the emotions in the same group*. If ($r \approx 0$) that means that vector emotions do not have a linear relationship, it is pos- sible to interpreted this as: *those vector emotions are not in the same group*.

Diagonal of Figure 4.8 shows the histogram corresponding to each vector emotion, that is, it shows the frequency of a value of each component vector, for example: vector emotion Joy has 5 components of value 1, 1 component of value 2, and so on. The line over the histogram is a density plot that shows the distribution of the data. Graphs below diagonal of the Figure 4.8 show the relation between vector emotions, and the red line in these graphs is a linear

regression which shows the tendency of the graphs. If the points are close to the red line then the emotions involved have a linear regression determined by the Pearson Coefficient (Hall, 2015).

In this way, each graph under diagonal correspond to a Pearson coefficient over the diagonal, for example, observing Figure 4.8 as a matrix, graph in position (2,1) (line, column) involving vector emotions “*Enjoy, Joy*” correspond to the Correlation Pearson Coefficient (1,2) with value 0.60. As this value is relative close to one, it is possible to say that these vector emotions have a “good” correlation. It is possible to observe this correlation between others emotions, as:

- *Happiness-Joy*;
- *Contempt, Irritation, Sadness, Fear, Grief, Anxiety, Anger, Pride*;
- *Shame-Guilt*;

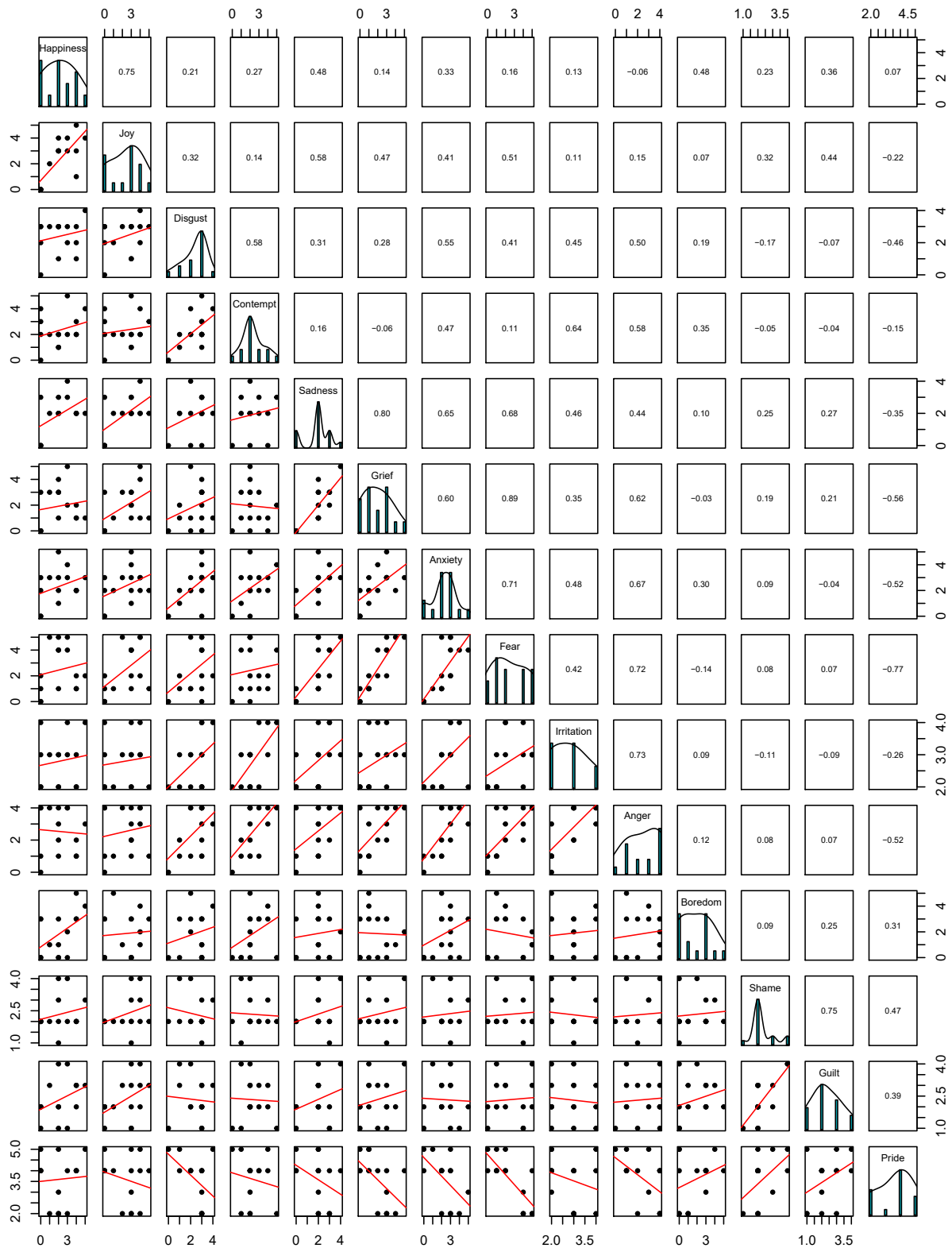


Figure 4.8: Scatter plot of the vector emotions (Scherer's data)

Until now, it is used a first approximation of the relation between vector emotions. Now, it is established another way to understand this “good” correlation between vector emotions using

PCA. Algorithms of R to implement PCA techniques are taken from (Kassambara, 2017).

Figure 4.9 explains percentage variation of each Principal Component by the lines *Proportion of Variance* and *Cumulative Proportion*. In *Proportion of Variance* is observed percentage variation for each component, for example: for the first component PC1 0.3886 (38.86%) of the variation information goes in that direction.

In general, it is used the first 2,3 or 4 Principal Components that contain almost all the information variation (Jolliffe, 2002). In this case, in the line *Cumulative Proportion* is observed that for the fourth component has a *Cumulative Proportion* of 0.81803, that number is the sum of the *Cumulative Proportion* of the above Principal Components. For that reason, in this study, it will be taken 4 Principal Components with a *cumulative proportion* of 0.81803 equivalent to 81.80 % of the information variation preserved by these 4 Principal Components.

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
standard deviation	2.3324	1.6608	1.4159	1.11776	0.86970	0.79069	0.6349	0.59579
Proportion of Variance	0.3886	0.1970	0.1432	0.08924	0.05403	0.04466	0.0288	0.02535
Cumulative Proportion	0.3886	0.5856	0.7288	0.81803	0.87206	0.91671	0.9455	0.97087
	PC9	PC10	PC11	PC12	PC13	PC14		
standard deviation	0.42551	0.32575	0.31359	0.11505	0.08873	0.03556		
Proportion of Variance	0.01293	0.00758	0.00702	0.00095	0.00056	0.00009		
Cumulative Proportion	0.98380	0.99138	0.99840	0.99935	0.99991	1.00000		

Figure 4.9: Principal Components with percentage variation

To choose the quantity of Principal Components is addressed by different methods. The problem with these methods is that its accuracy depends on what is looking for in using PCA, that is, there is not an only truth about how to calculate the quantity of

(Principal Component Analysis (PCA) is a mathematical algorithm that study the variation of a data set reducing the dimensionality of the data (Ringnér, 2008).) components, for example, in some cases it is needed a *cumulative proportion* of 99.99% to capture data variance, in some cases it is needed 70% because it is important just the visualization of data (Bro and Smilde, 2014). For that reason, here it is explained two of the most popular methods according to (Bro and Smilde, 2014):

- Scree Test;
- Eigenvalue below one or Kaiser - Guttman's Rule

Scree Test is a visual test and the idea is to take components until there is no variation of one component with respect to the next component (Bro and Smilde, 2014). For example, Figure 4.10 shows how the difference between fifth and sixth components are low ($5.4\% - 4.5\% = 0.9$), but the same is possible to say with respect to Components 7 and 8 in which its difference is ($2.9\% - 2.5\% = 0.4\%$). In this way, it is possible to guess that it is necessary just 5 or 7 Principal Components.

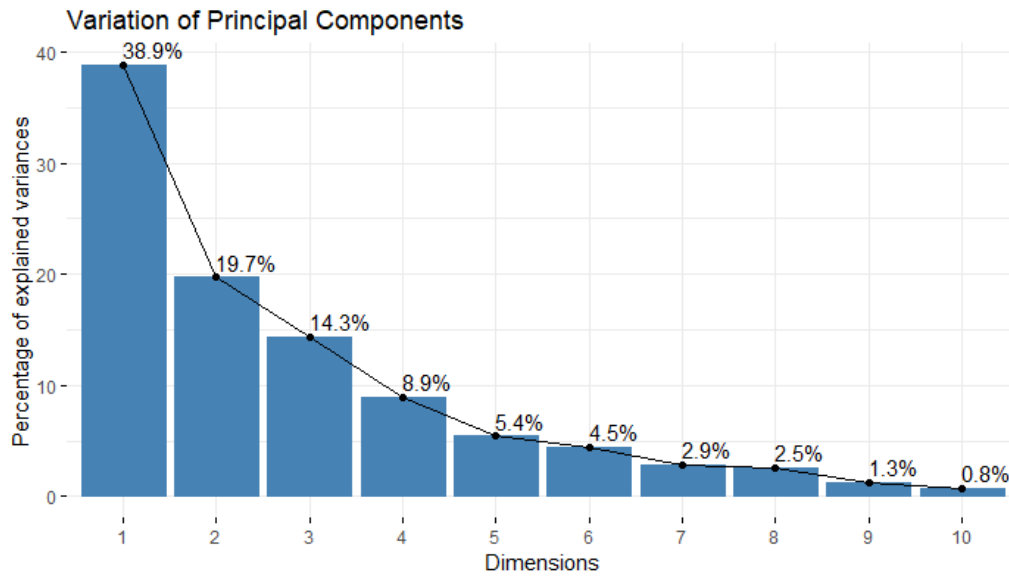


Figure 4.10: Variation of data with respect to each principal component

Eigenvalue below one is a method in which consider just the Principal Components such that its eigenvalues associated are below one (Bro and Smilde, 2014). For example, Figure 4.11 shows that fifth eigenvalue is below one, then, with respect to this test, it is necessary just to take 4 Principal Components.

	eigenvalue	variance.percent	cumulative.variance.percent
Dim.1	5.440000366	38.857145474	38.85715
Dim.2	2.758206837	19.701477410	58.55862
Dim.3	2.004834086	14.320243471	72.87887
Dim.4	1.249387108	8.924193627	81.80306
Dim.5	0.756376620	5.402690145	87.20575
Dim.6	0.625198538	4.465703846	91.67145
Dim.7	0.403153492	2.879667800	94.55112
Dim.8	0.354959995	2.535428532	97.08655

Figure 4.11: Values of a few eigenvalues associated to its respective Principal Components

In this way, it is chosen 4 Principal Components according to the *Eigenvalue below one test*, which is more accurate with respect to *Scree Test*, in this case. 4 Principal Components have a *Cumulative Proportion* of 81.80% preserving data variance. This study will use 4 Principal Components, it is not possible to represent a space of 4 dimensions, for that reason it will be presented the relation between principal components by pairs, as presented in Figures 4.12, 4.13, 4.14, 4.15, 4.16 and 4.17.

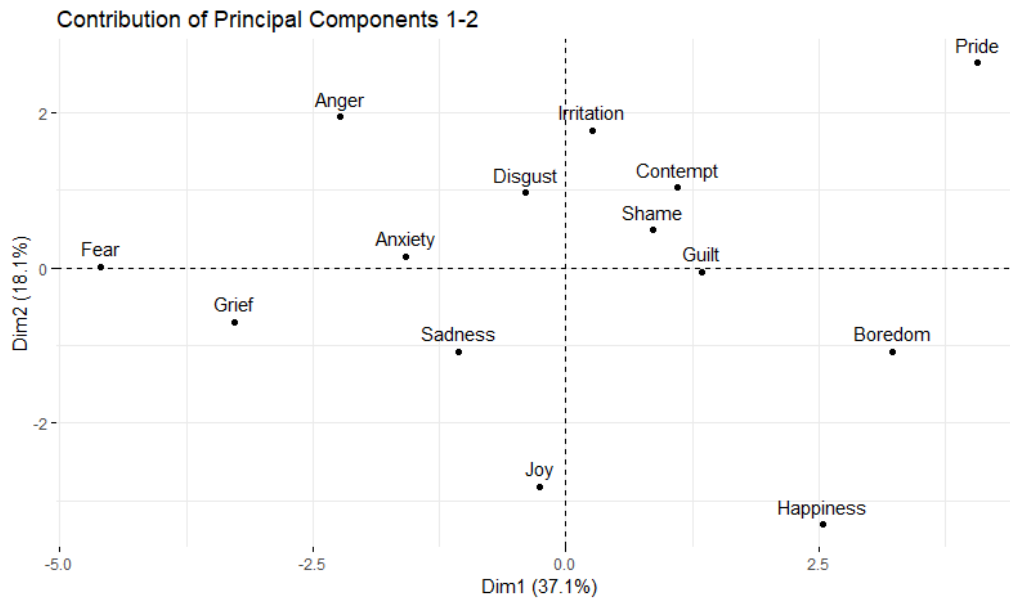


Figure 4.12: Principal Components 1 vs 2

It is possible to observe in Figure 4.12 how positive emotions as *Joy*, *Happiness* are separated from negative emotions. Also, it is possible to observe how an emotion as *Pride* is isolated by these two principal components.

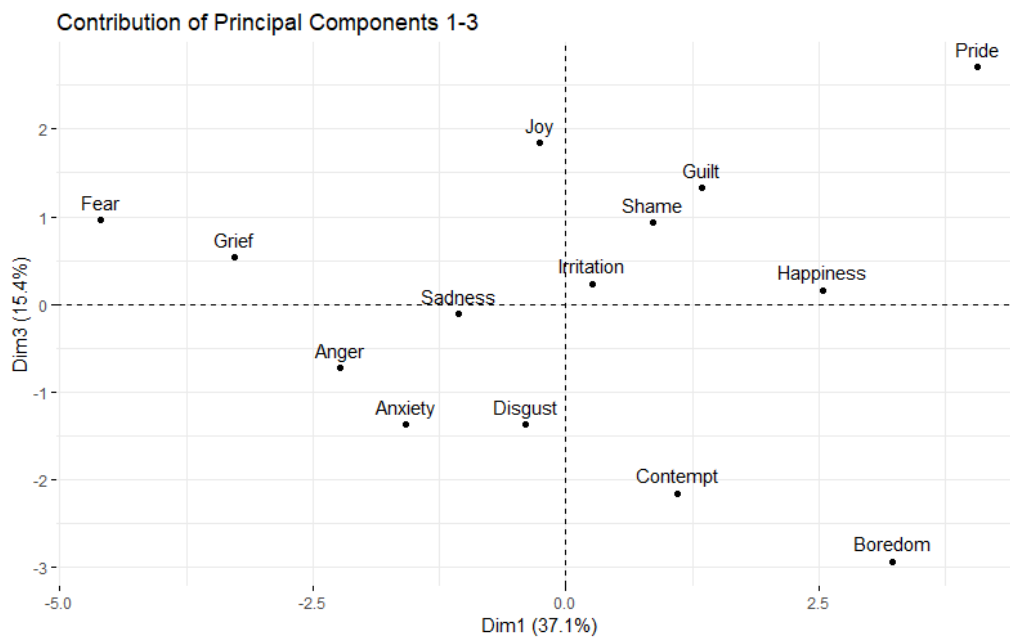


Figure 4.13: Principal Components 1 vs 3

Figure 4.13 shows how positive emotions are influenced by different Principal Components.

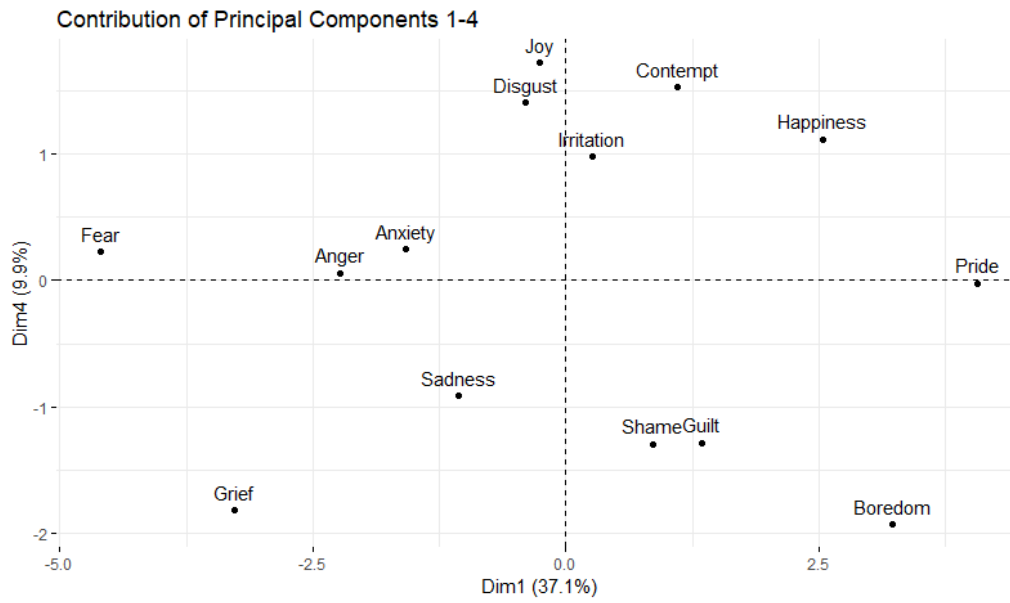


Figure 4.14: Principal Components 1 vs 4

Figure 4.14 shows how positive emotions as *Joy* has proximity with emotions as Disgust, given by principal Component 1.

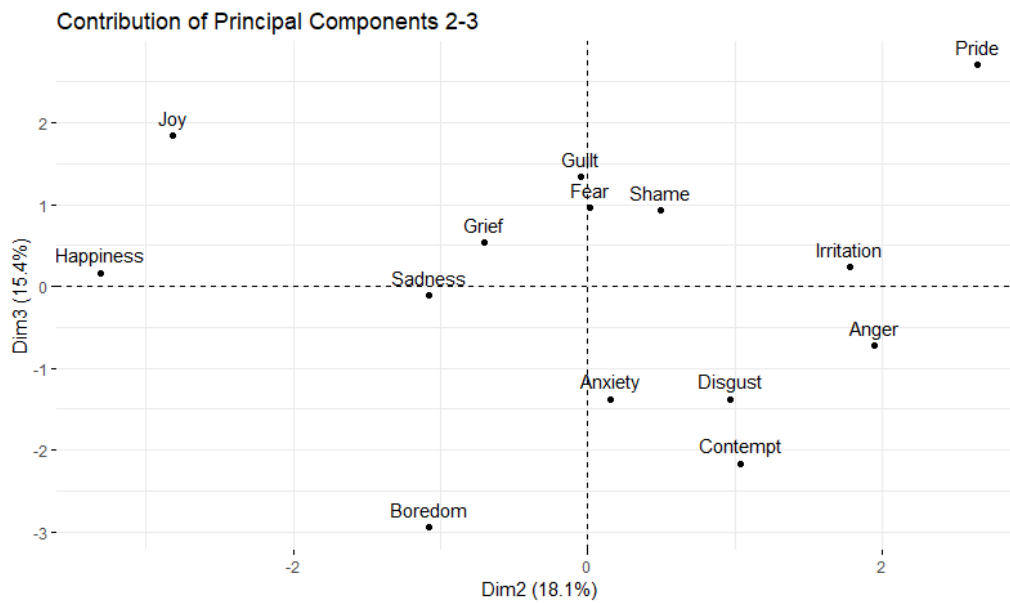


Figure 4.15: Principal Components 2 vs 3

Figure 4.15 shows how Principal Components 2 and 3 separate positive emotions from negative emotions and again *Pride* as a isolated point.

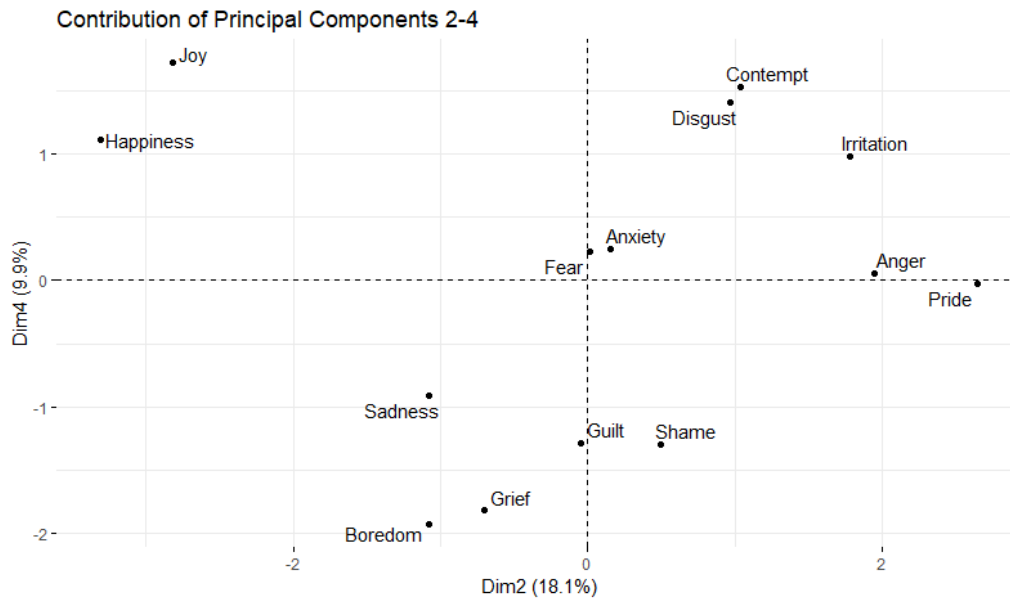


Figure 4.16: Principal Components 2 vs 4

Figure 4.16 shows how Principal Componentes 2 and 4 separate positive emotions from negative emotions with the difference that *Pride* emotion is close to *Anger* emotion.

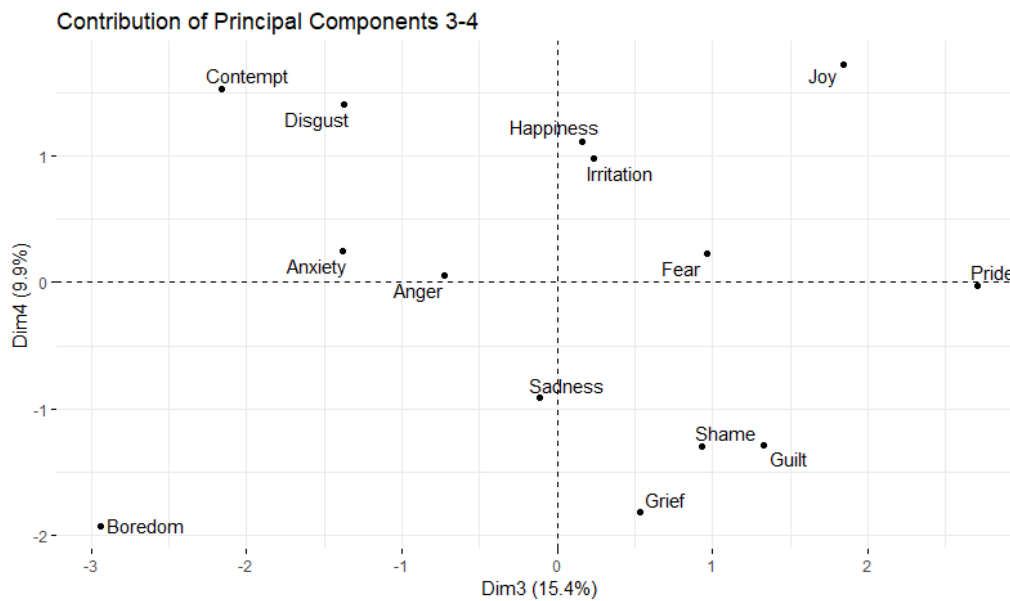


Figure 4.17: Principal Components 3 vs 4

Figure 4.17 shows a proximity between positive emotion *Happiness* with *Irritation* emotion consider as negative.

It is possible to observe in Figures 4.13, 4.14 and 4.17 how some vector emotions are not expected to have proximity between them as the case of vector emotion *Joy* in Figures 4.13 and 4.14, and the case of the vector emotion *Happiness* in Figure 4.17 that are close to vector emotions consider as negative emotions. Also, it is possible to observe in Figures 4.12, 4.15, 4.16 how positive emotions (*Joy*, *Happiness*) are separated from negative emotions given by

pairs of Principal Components (1,2), (2,3) and (2,4). Proximity between emotions and separation between them can be explained by the influence of certain variables associated to each emotion, that is, it is possible that certain positive emotions share some variable that elicit negative emotions also.

For that reason it is established a *Cluster* using the algorithm K-means. This cluster is compared with *Pearson Coefficient* showed in Figure 4.8 to establish definite groups of emotions.

4.3.1 Data Clustering Analysis

In this study data is very small, and it is possible to study with a few tools as K-means algorithm, which is the most popular algorithm for classification of data, but also it is a simple method (Kodinariya and Makwana, 2013). In this research It is chosen k-means because the algorithm establishes groups based on a metric, in this case the euclidean metric. Vector emotions with less euclidean distance are more close to each other.

K-means Algorithm

According to (Kodinariya and Makwana, 2013):

The main idea is to define N centroids, one for each cluster. These centroids should be placed in a cunning way because of different location causes different result. So, the better choice is to place them as much as possible far away from each other. The next step is to take each point belonging to a given data set and associate it to the nearest centroid. When no point is pending, the first step is completed and an early group is done. At this point, it is needed to re-calculate N new centroids as centers of the clusters resulting from the previous step. After these N new centroids, a new binding has to be done between the same data points and the nearest new centroid. A loop has been generated. As a result of this loop it may notice that the N centroids change their location step by step until no more changes are done. In other words centroids do not move any more.

K-means responds to an objective function which it is necessary to minimize showed in equation 4.3. In equation 4.2, C represents *Sum* of distances from a set of points y_i to a centroid c_k . Each point y_i is in a cluster S_k . C represents proximity of points of a cluster to its centroid.

In equation 4.3, $W(C)$ represents *Sum* of the distance C for each cluster S_k . N represents the total quantity of cluster.

$$C = \sum_{i \in S_k} \|y_i - c_k\|^2 \quad (4.2)$$

$$W(C) = \sum_{k=1}^N \sum_{i \in S_k} \|y_i - c_k\|^2 \quad (4.3)$$

The first step is to find the ideal quantity of cluster N such that minimize $W(C)$. For that, there are a variety of methods to find this ideal quantity. In this work it is explore a few methods (more popular and simple) because as in PCA there is not just one method to find the ideal quantity of clusters. It is chosen the most popular and simple because data Scherer is small, so, it is not necessary a sophisticated algorithm to find a relation in Scherer's data. These methods are taken from (Kodinariya and Makwana, 2013):

- Rule of thumb;
- Elbow method;
- Silhouette Method.

By *rule of thumb method* is a simple calculation (Kodinariya and Makwana, 2013). In this case, n is the number of data points, that is 14 (emotions):

$$N \approx \sqrt{\frac{n}{2}} \quad (4.4)$$

$$N \approx \sqrt{\frac{14}{2}} \quad (4.5)$$

$$N \approx 2.65 \quad (4.6)$$

$$N = 3. \quad (4.7)$$

In this way, to *rule of thumb method* it is necessary 3 clusters.

In the moment of implement k-means in software R with a quantity of clusters $N = 3$ it is obtained the results exposed in Figure 4.18. In this figure, it is observed how *Pride* Emotion is consider as a negative emotion against Scherer's point of view in (Scherer et al., 2001). For that reason it is not consider $N = 3$ as the number of clusters.

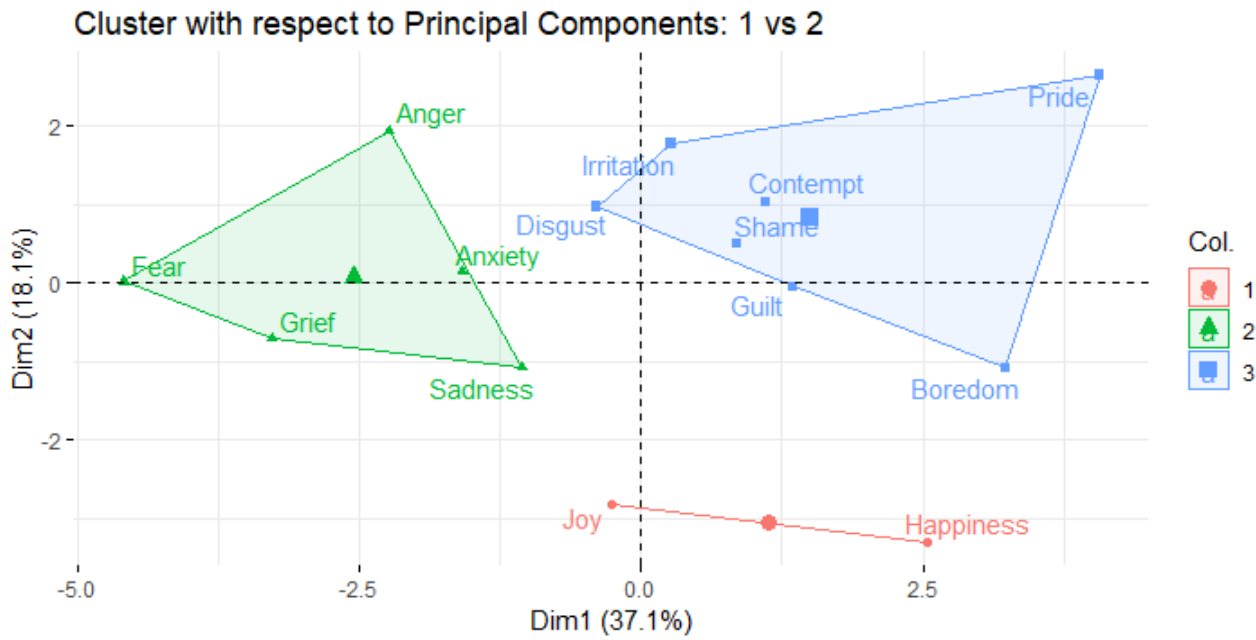


Figure 4.18: Clusters with $N = 3$ - Pride as a negative emotion

Elbow method is a *visual* method, similar to *Scree test* from PCA, because the idea is to make a plot of the quantity of cluster N *vs* the value of $W(C)$ (see Figure 4.19) and choose a quantity N of clusters observing if the variation of the $W(C)$ with respect to next value of $W(C)$ is low. In Figure 4.19 is possible to observe that in $N = 8$ the variation of $W(C)$ value with respect to next one is low. Then, in this case, it is said that the quantity of cluster is 8, according to *Elbow method*.

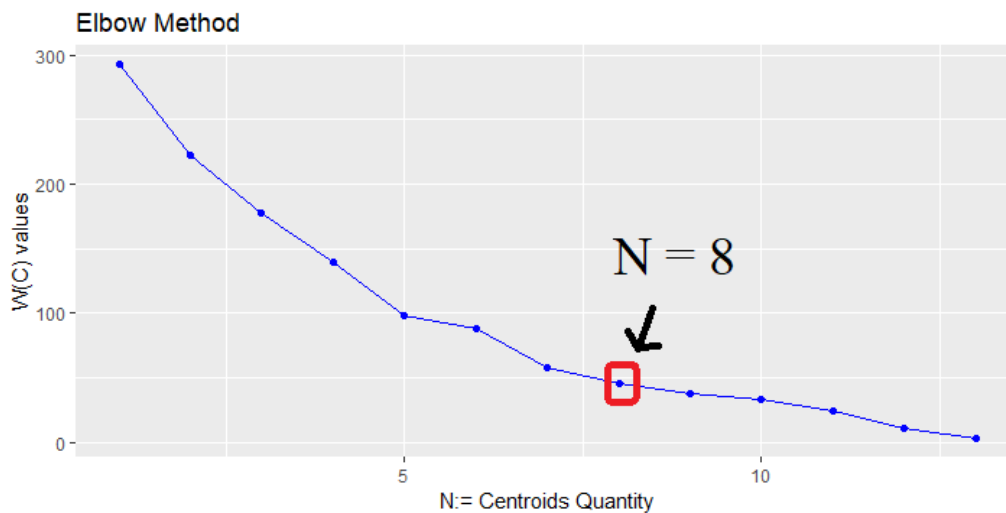


Figure 4.19: Quantity of Clusters N *vs* $W(C)$ values

In the moment of implement in software R using clusters $N = 8$ it is obtained next results showed in Figure 4.20. It is observed that *Fear* emotion is isolated from *Anxiety*, *Anger Emotion*. In Scherer et al. (2001), *Anxiety* and *Fear* have a strong bond because the multiple per-

ception of the future are tied to Fear, for that reason it is not consider $N = 8$ as a good number of clusters.

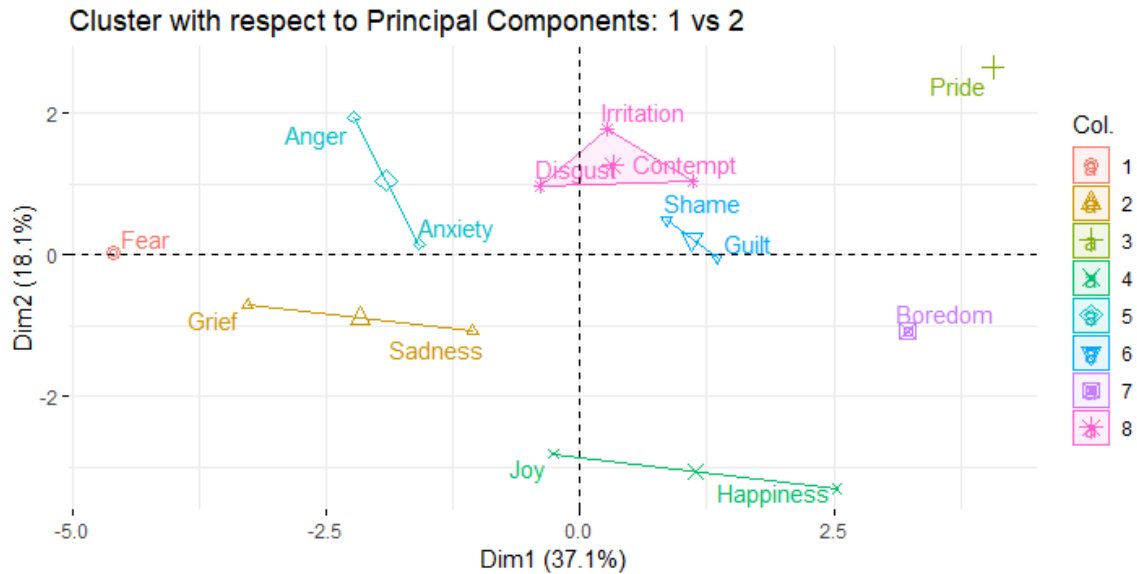


Figure 4.20: Clusters with $N = 8$ - Fear not related to *Anxiety* emotion

Silhouette Method is a method that measures distance between clusters depending of the selection of Cluster quantity N . The optimal quantity of clusters N depends of this distance between cluster, that is, highest distance between clusters corresponds to the optimal quantity of clusters N . Results are obtained in Figure 4.21 showing that optimal quantity of clusters is $N = 4$.

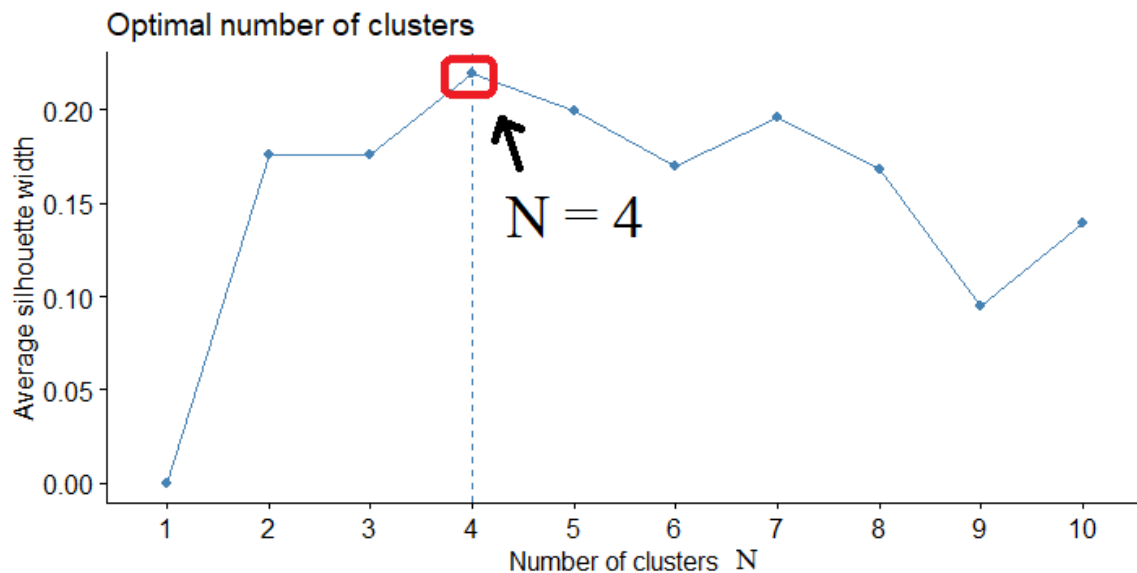


Figure 4.21: Silhouette Method for K-means

In the moment of implement k-means with a quantity of clusters $N = 4$ it is obtained a reasonable result classifying emotions in *Positive* and *Negative*, *Pride* emotion is isolated,

neither positive or negative, *Anger* emotion is associated with *Fear* emotion according to (Scherer et al., 2001). Figure 4.22 shows the Clusters for $N = 4$.

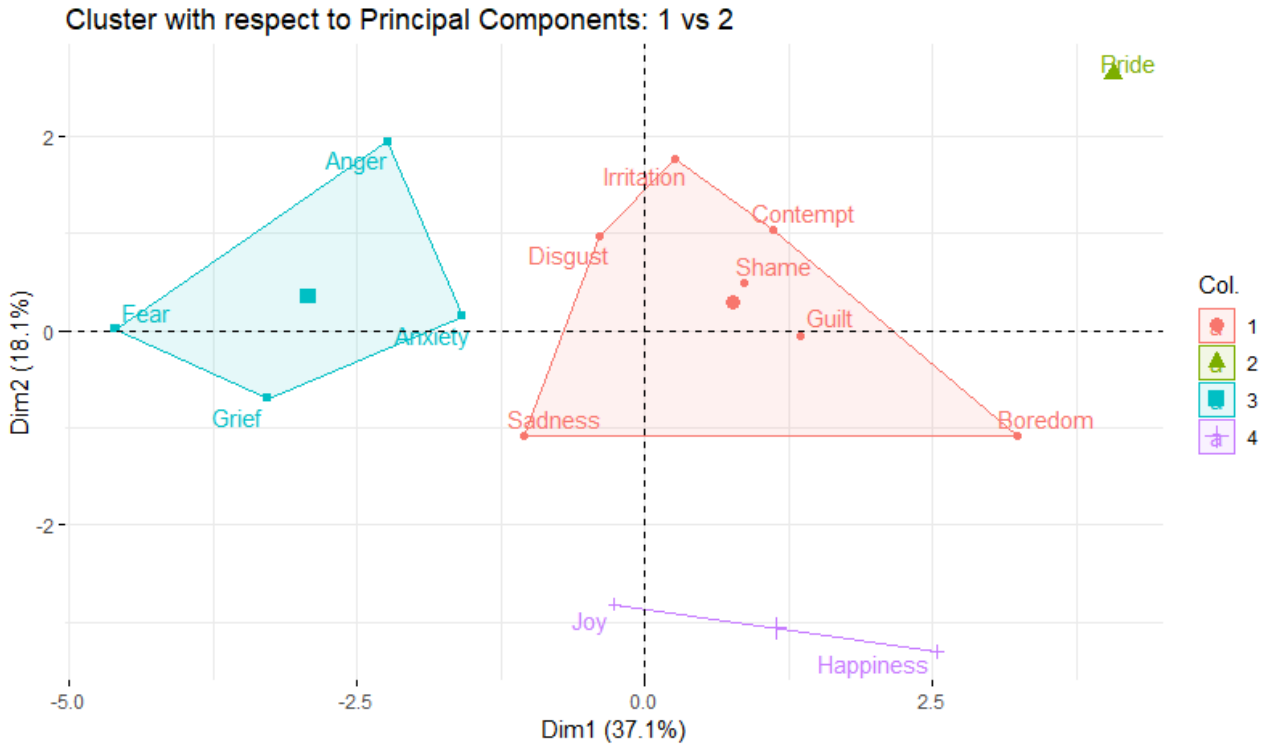


Figure 4.22: Cluster with respect to Principal Components 1 vs 2

Based in these results, in this work it is used a quantity of Clusters $N = 4$. Having these groups of emotions given by algorithm of clustering K-means:

- a) *Anger, Fear, Anxiety, Grief*;
- b) *Irritation, Contempt, Shame, Guilt, Boredom, Sadness, Disgust*;
- c) *Joy, Happiness*;
- d) *Pride*.

These groups are similar to the groups found with Correlation Pearson Coefficient (see groups (Figure 4.8).

K-means was develop with euclidean metric and it is contrasted with Correlation Pearson Coefficient which is Pearson metric. So, from two different metrics it is obtained similar groups. It means that some emotions are close to have a linear relation in the case of Correlation Pearson Coefficient and some emotions have similar variation in the case of k-means using PCA.

Now, with respect to PCA, the quantity of Principal Components is 4, then, it is not possible to visualize emotion groups in 4 dimensions, for that reason it is visualised in pairs of Components, as presented in Figures 4.23, 4.24, 4.25, 4.26 and 4.27.

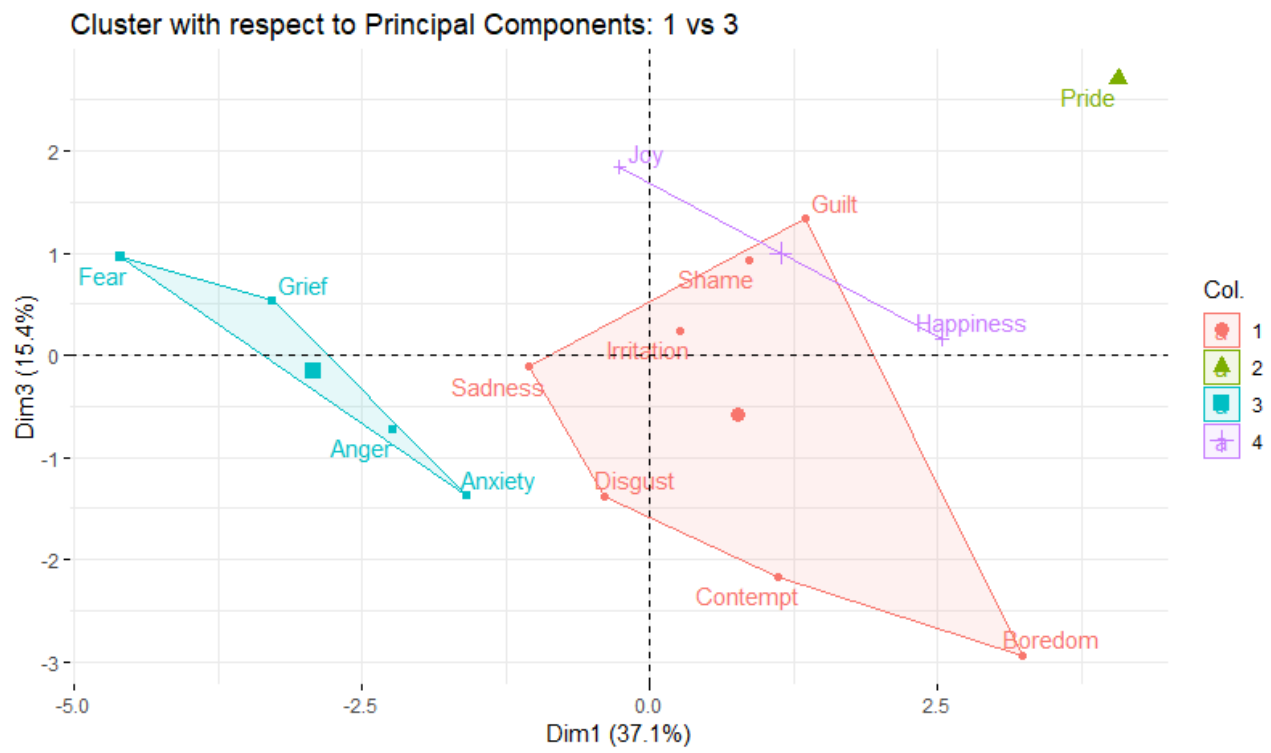


Figure 4.23: Cluster with respect to Principal Components 1 vs 3

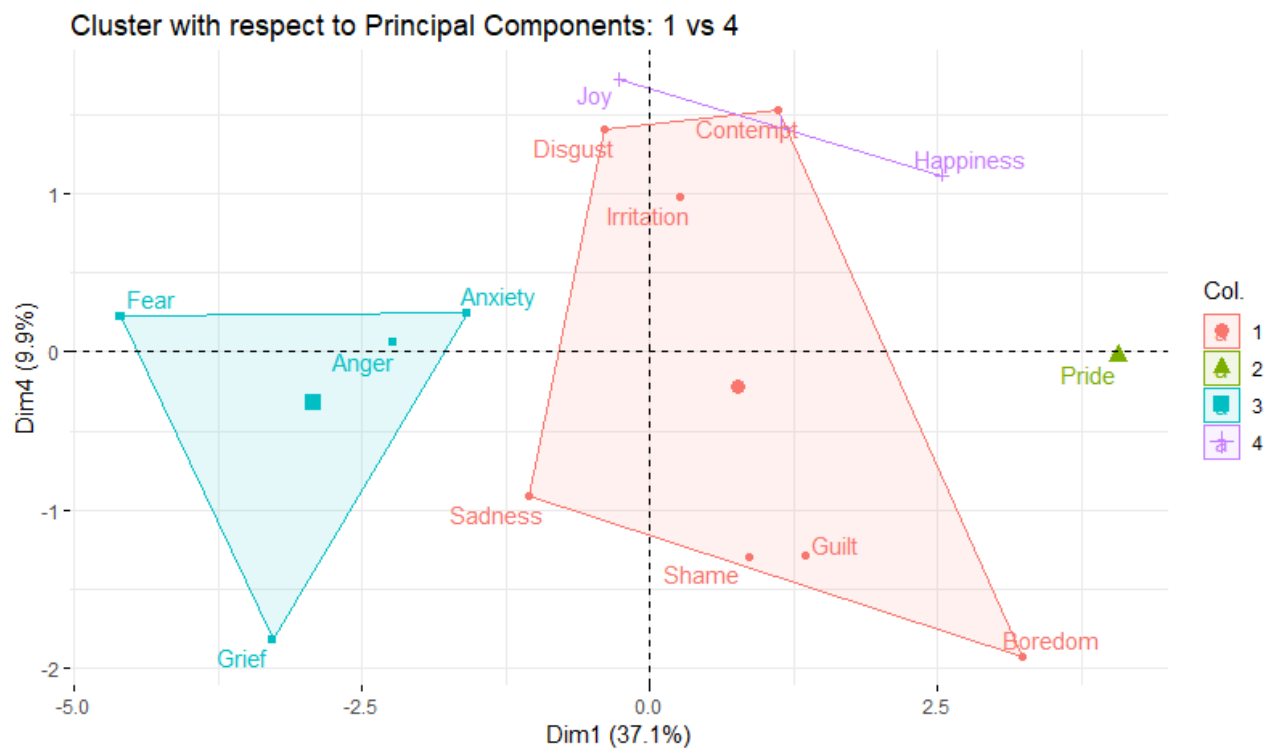


Figure 4.24: Cluster with respect to Principal Components 1 vs 4

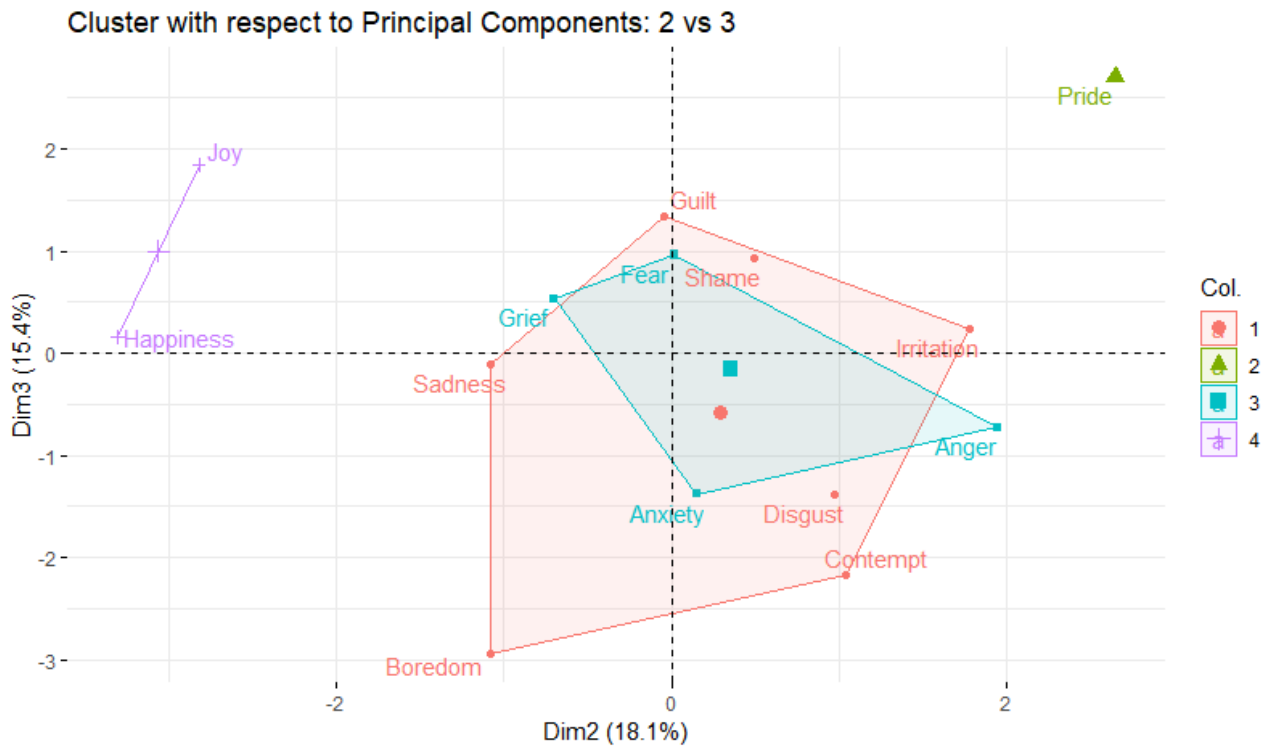


Figure 4.25: Cluster with respect to Principal Components 2 vs 3

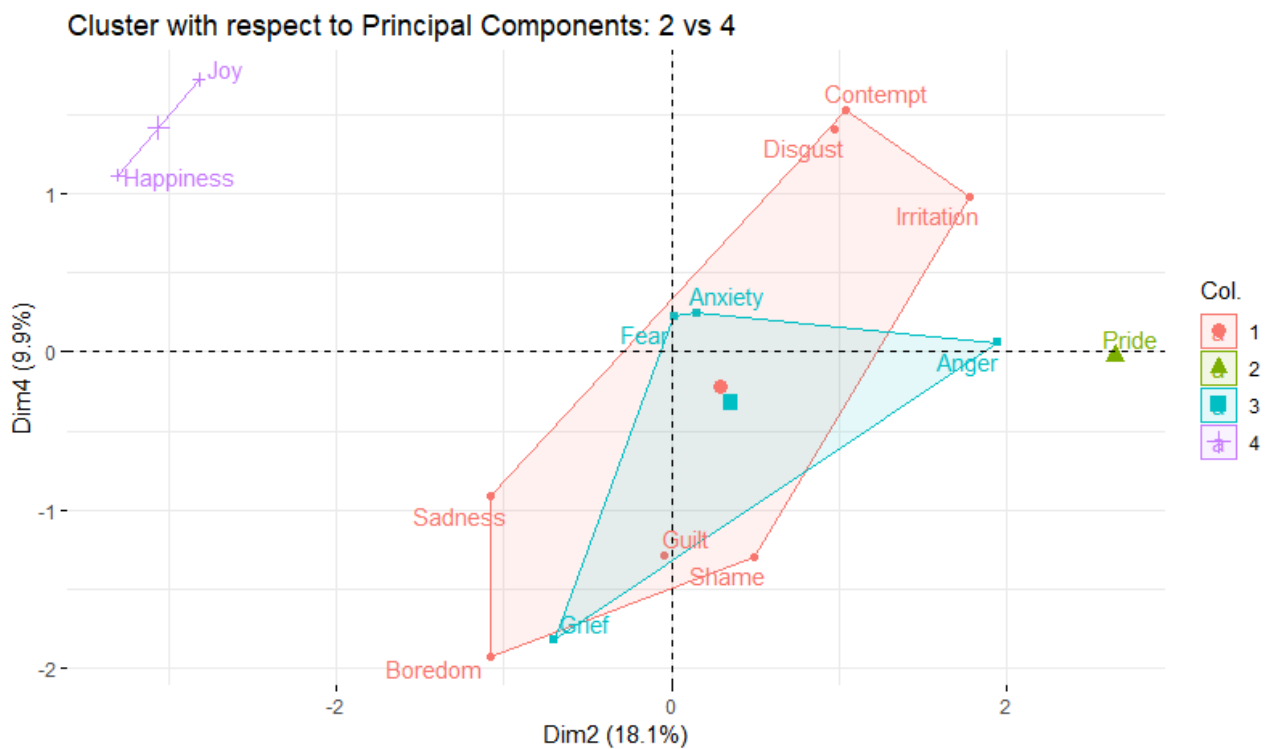


Figure 4.26: Cluster with respect to Principal Components 2 vs 4

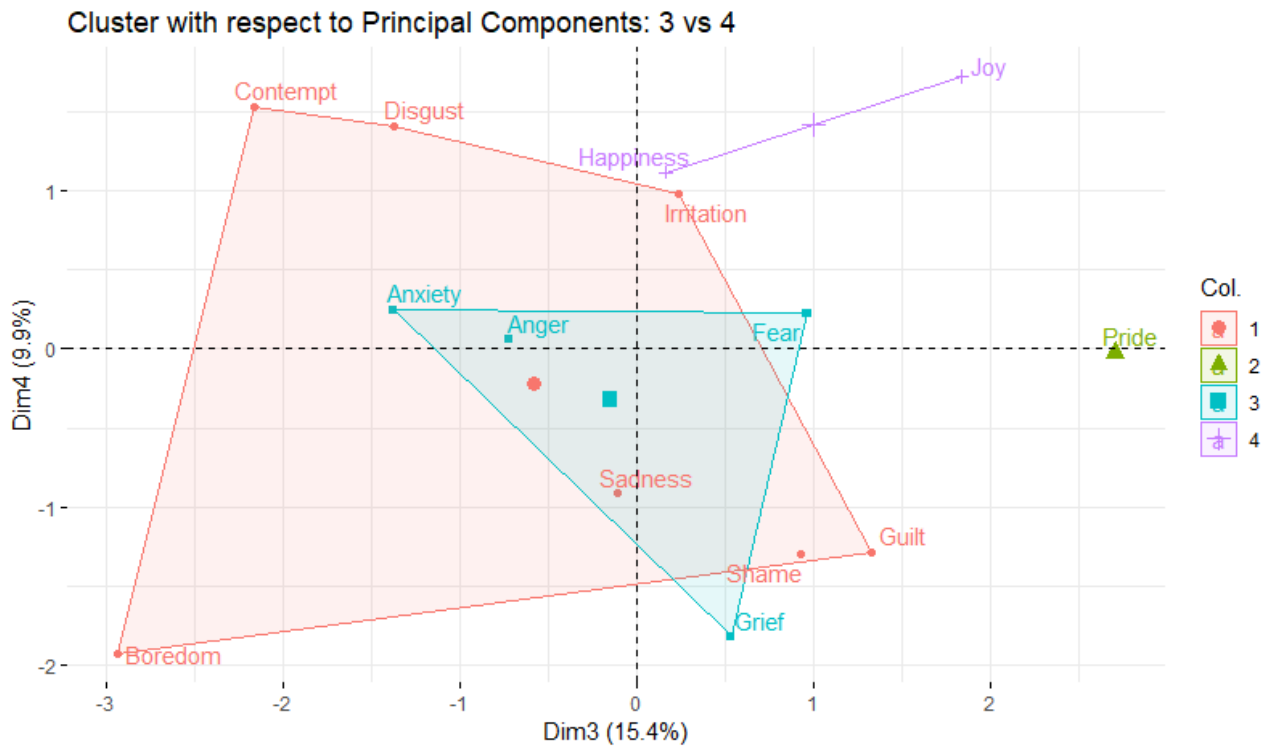


Figure 4.27: Cluster with respect to Principal Components 3 vs 4

It is possible to observe in Figures 4.23 and 4.24 the proximity of Positive Emotions to negative emotions with respect to some Principal Components. In Figures 4.25, 4.26 and 4.27 are observed the proximity between groups of negative emotions (groups of blue and red color) with respect to some Principal Components. A possible explanation of proximity is that these emotions are elicited by similar variables (checks) and PCA record similar variation of data.

About groups obtained in this classification, it is curious to observe that vector emotions: *Fear* and *Anxiety* are in the same group, it is curious because in (Scherer et al., 2001) *Fear* and *Anxiety* are highly correlated supported by clinical studies, considering *Fear* as the central emotion in *Anxiety*. Besides, in Scherer's study (Scherer, 1993), he uses the words *sadness* and *depression* as synonymous. So, it is possible to have a tentative names for these groups as:

- Anxiety Group;
- Depression Group.

In this way, it is possible to compare groups obtained from k-means and groups obtained from Pearson Correlation:

Pearson Correlation Groups

- Contempt, Irritation, Sadness, Fear, Grief, Anxiety, Anger, Pride*

- *Shame, Guilt*
- *Joy, Happiness*

K-means Groups

- Anger, Fear, Anxiety, Grief
- Irritation, Contempt, Shame, Guilt, Boredom, Sadness, Disgust
- Joy, Happiness
- *Pride.*

K-means Groups is more related to Scherer views about emotions because he does not label Pride emotion as a negative emotion. For that reason is choose k-means group as the definite group to continue with the study.

Now, having groups emotions it is possible to establish a relation between groups using Category Theory (CT) and Scherer's theory of emotions explained in section 2.

In this way, next chapter explains basic notions of CT and it is implemented groups of emotions (Scherer groups founded in this section and OCC groups of emotions given by the theory itself) as categories (in a mathematical sense).

5 CATEGORY THEORY AND APPRAISAL THEORIES

In this chapter, the emotions model from Scherer (1993) and emotions from OCC are modelled basing to categories.

5.1 Categories for Scherer Emotions

After presenting the intuition of a Category and a Functor, the Figure 5.1 illustrates the structure of Appraisals in Scherer's theory. Appraisals in Scherer's theory works in a sequence. When an emotion emerges to a subject, *Relevance* Appraisal is the first in to be activated, after that, comes the appraisal *Implication* and so on. Those four Appraisals constitute an Emotion, a category, for that reason in Figure 5.1 presented arrows as a composition of arrows to satisfy the definition of a category. These objects are *Relevance*, *Implication*, *Coping*, *Normative Significance* and the arrows are $f, g, h, f \circ g, h \circ g$. These arrows establish the order relation between the Appraisals *Relevance*, *Implication*, *Coping Potential*, *Normative Significance*, that is: f establishes the order relation between *Relevance* and *Implication*, g establishes the order relation between *Implication* and *Coping Potential*, h establishes the order relation between *Coping Potential* and *Normative Significance*. The compositions $g \circ f, h \circ g$ are used just to satisfied the definition of a category. It is necessary to explain that the four groups of emotions used are obtained by the K-means algorithm. It is chosen the groups of K-means because with respect to the vector emotion **Pride**.

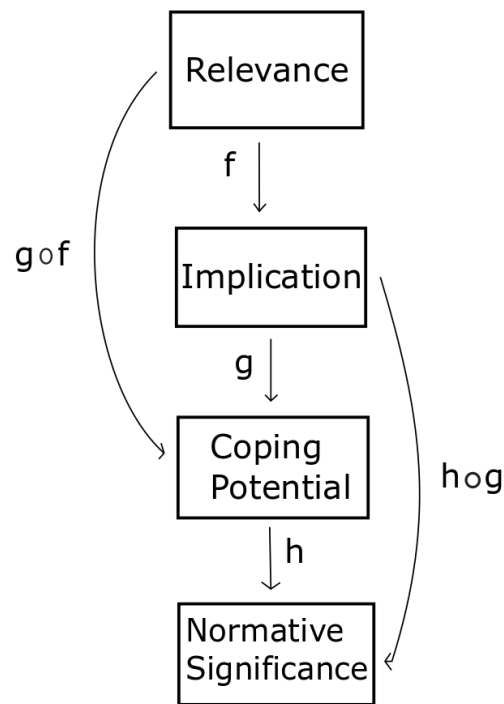


Figure 5.1: Scherer's sequences of Appraisals presented with composition between arrows

Figure 5.2 presents the elements of objects of the category, that is, the elements (*Checks*) of the appraisals. For example, in the object (Appraisal) *Relevance* there are the elements: *Novelty*, *Intrinsic Pleasantness*, *Goal Relevance*; and in the object (Appraisal) *Implication Assessment* there are the elements: *Probability*, *Discrepancy from expectation*, *Goal/Need Conduciveness*, *Urgency*.

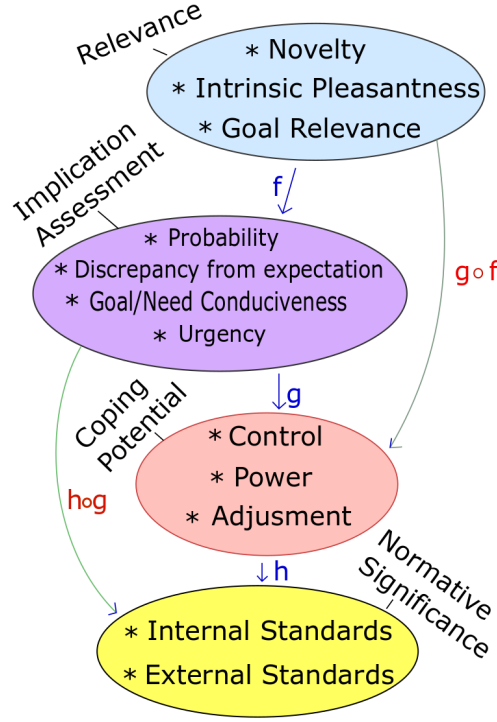


Figure 5.2: Scherer's Emotion General Model as a Category, adding Checks for each Appraisals

Finally, Figure 5.3 shows the relation between Categories (emotions) through a Functor. In this figure there are the categories: *Emotions 1*, *Emotion 2* (in red color) and the Functor H (in brown color). It is necessary to remember that the functor H in Figure 5.3 carries images of each object of the category, in this case: *Relevance*, *Implication Assessment*, *Coping Potential*, *Normative Significance*, each one of them has image through the Functor H , that is:

$$H(Relevance_{emotion\ 1}) = Relevance_{emotion\ 2} \quad (5.1)$$

$$H(Implication\ Assessment)_{emotion\ 1} = Implication\ Assessment_{emotion\ 2} \quad (5.2)$$

$$H(Coping\ Potential_{emotion\ 1}) = Coping\ Potencial_{emotion\ 2} \quad (5.3)$$

$$H(Normative\ Significance)_{emotions\ 1} = Normative\ Significance_{emotion\ 2} \quad (5.4)$$

Looking to Figure 2.5, it is possible to understand the general idea.

Functor in this case establish the relation obtained by k-means. For example, one of the groups establish by k-means is *Positive Emotions: Happiness and Joy*. A Functor establish the relation between these emotions given by the k-means.

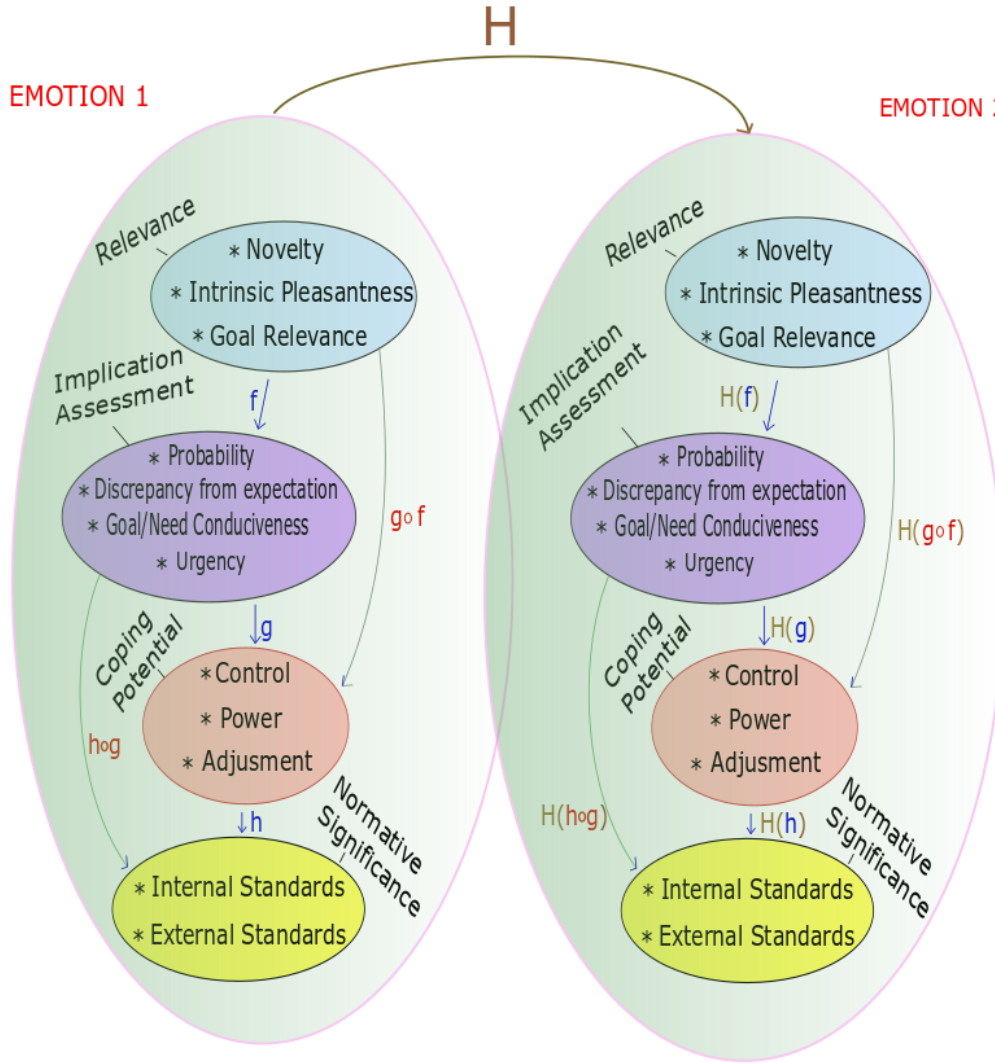


Figure 5.3: Relation between categories (emotions) through a Functor

Once established emotions as categories and the relation between them through a Functor, it is used the values from Figures 4.1, 4.2, and the results presented in Figures 4.22, 4.23, 4.24, 4.25, 4.26 and 4.27, to define quantitatively the categories (emotions) and the relation between them through a Functor ³.

³*Probability Check* has a value of “?”, because in data from (Scherer, 1993) it is not assigned any value because *Probability Check* does not appear as a *Check Appraisal*, but in (Scherer et al., 2001) *Probability Check* appear as a *Appraisal Check*, for that reason was included in this proposal.

It is chosen 4 groups of vector emotions based on the Scherer's theory taken from (Scherer et al., 2001) and the different way to choose the quantities of groups (k) that offers the algorithm.

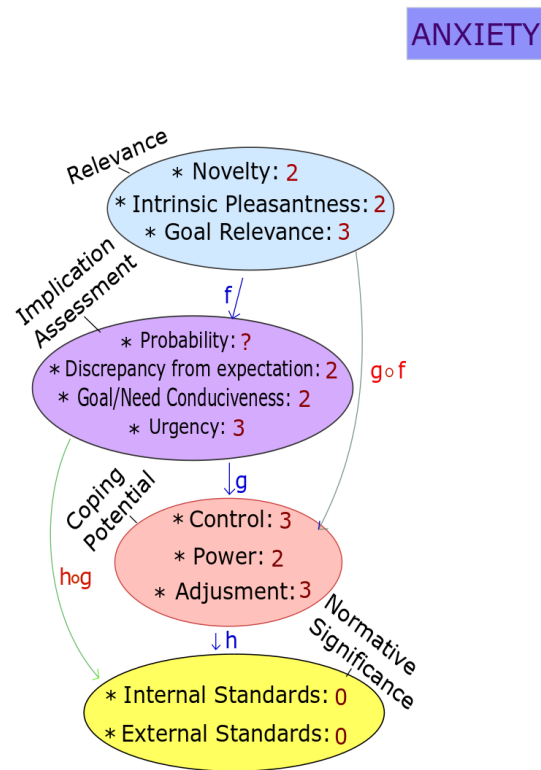


Figure 5.4: Anxiety values in its category

FEAR

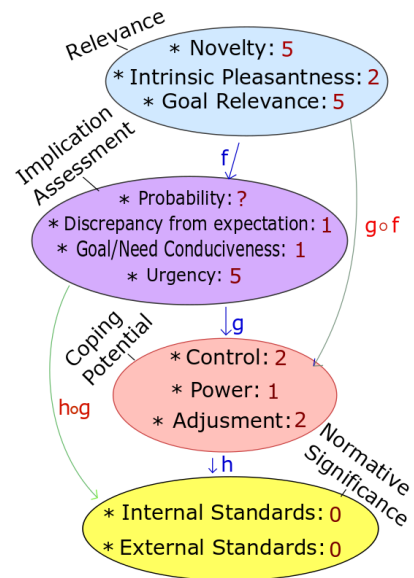


Figure 5.5: Fear values in its category

ANGER

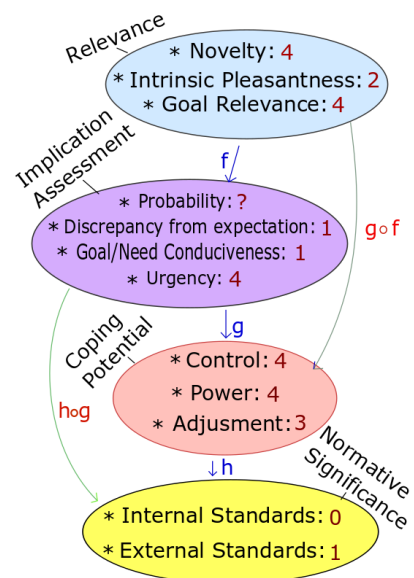


Figure 5.6: Anger values in its category

GRIEF

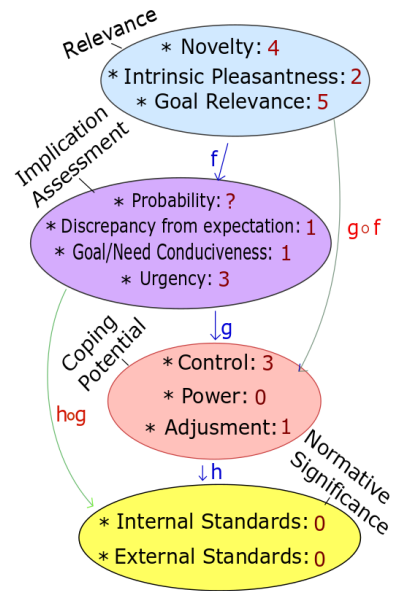


Figure 5.7: Grief values in its category

CONTEMPT

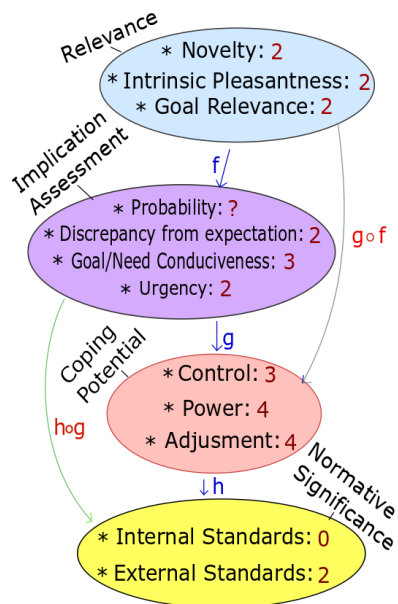


Figure 5.8: Contempt values in its category

DISGUST

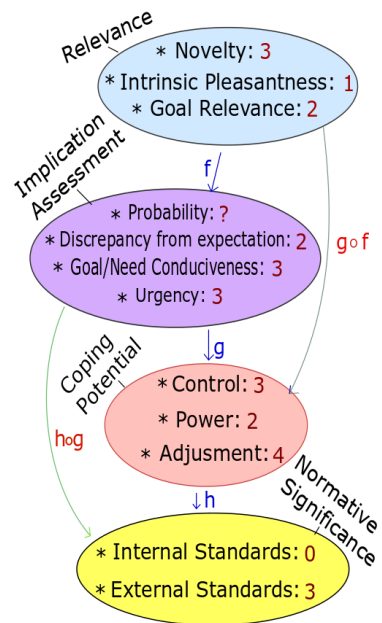


Figure 5.9: Disgust values in its category

IRRITATION

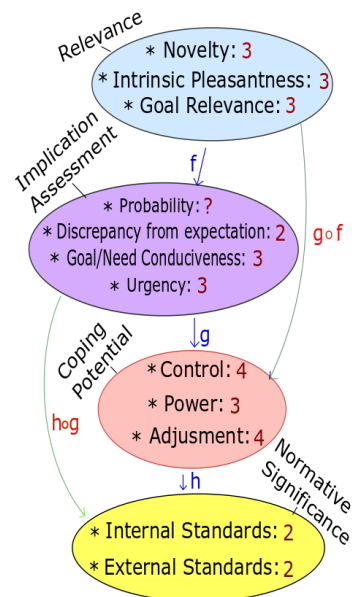


Figure 5.10: Irritation values in its category

SHAME

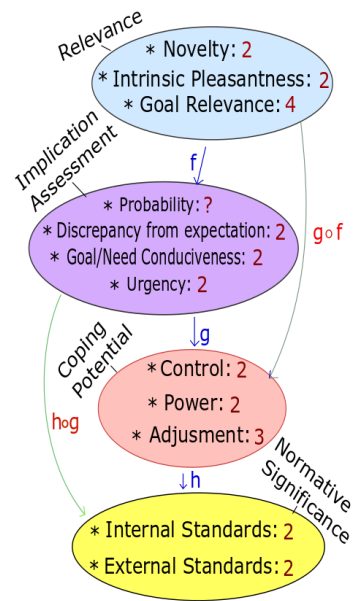


Figure 5.11: Shame values in its category

GUILT

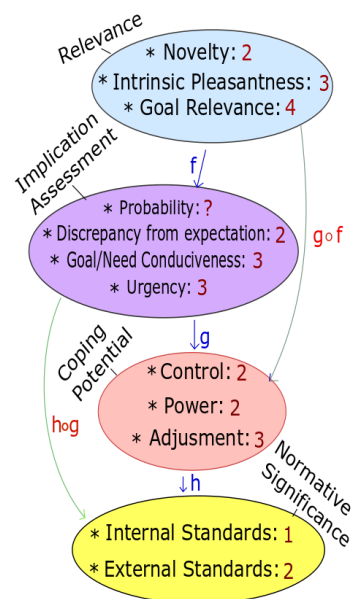


Figure 5.12: Guilt values in its category

BOREDOM

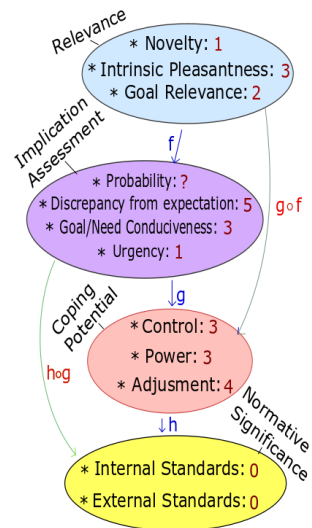


Figure 5.13: Boredom values in its category

SADNESS

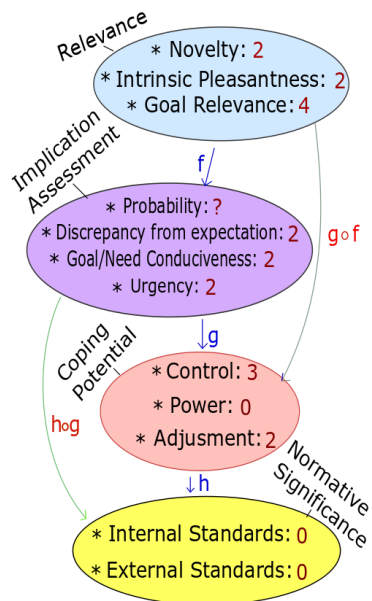


Figure 5.14: Sadness values in its category

JOY

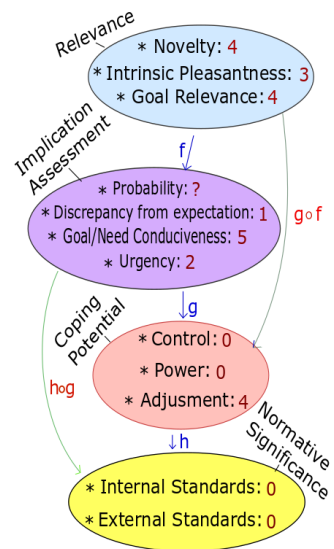


Figure 5.15: Joy values in its category

HAPINESS

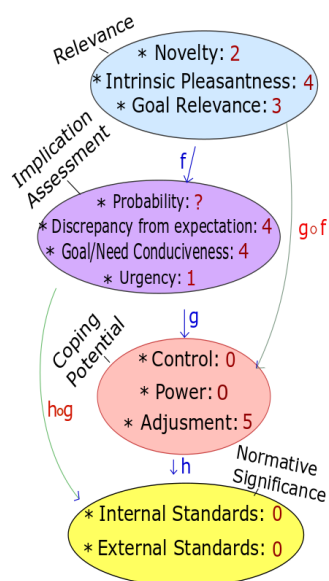


Figure 5.16: Happiness values in its category

The last emotion observed as a category is *Pride*, as shows Figure 5.17. Also, this figure represents the first group of the clustering develop with the algorithm *k-means*. The next step is to establish the connection between emotions of the same group with the notion of Functor. In this way, Figure 5.18 represents the group of emotions composed of just one emotion.

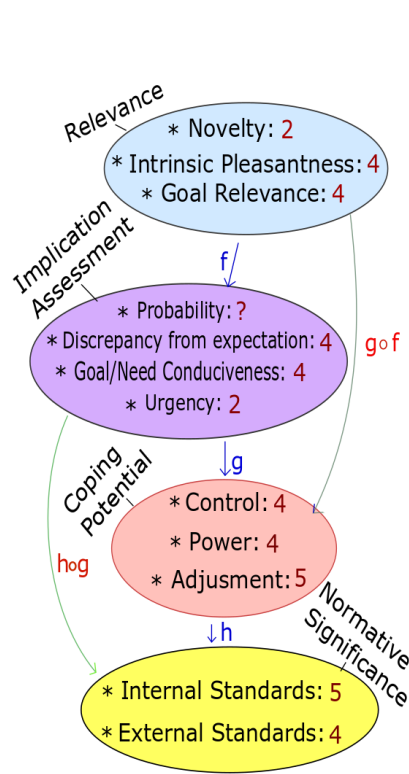


Figure 5.17: Pride values in its category

Figure 5.18 exhibits the second group composed of four emotions named as the *Anxiety* Group because of the relation between *Anxiety* and *Fear* expressed in (Scherer et al., 2001).

In Figures 5.18 and 5.19 it is not presented the names of the *Checks* to keep clarity in the images, but names of each emotion is written. Arrows in brown: $F1$, $G1$, $H1$, $J1$ represents the Functors that express the relation between Categories (Emotions).

Functors express the relation established by k-means, that is, each group in k-means is represented by the category of each emotion and the relations between categories given by the Functors.

Definitions of Functors are like shows figure 5.3 taking into account that each object (*Relevance*, *Implication Assessment*, *Coping Potential*, *Normative Significance*) is mapped in itself as shows equations: 5.1, 5.2, 5.3, 5.4

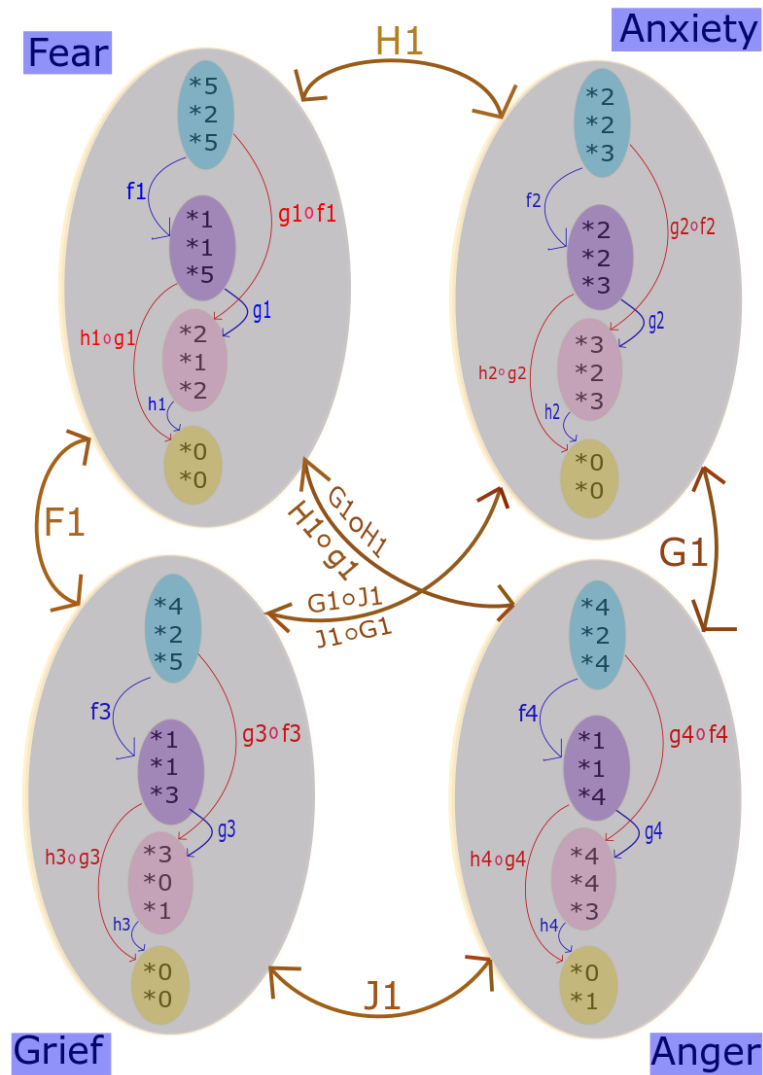


Figure 5.18: Relation between categories (Emotions) in the *Anxiety* group

In Figure 5.19 shows other group of emotions vector named as *Depression* group. It is not represented all the arrows in brown color (Functor) between emotions (categories) because the figure could result a little confused, but the idea is that all the emotions are related by an arrow in brown color (Functor), that is to represent the relation of the emotions as a group and to express the notion of a category.

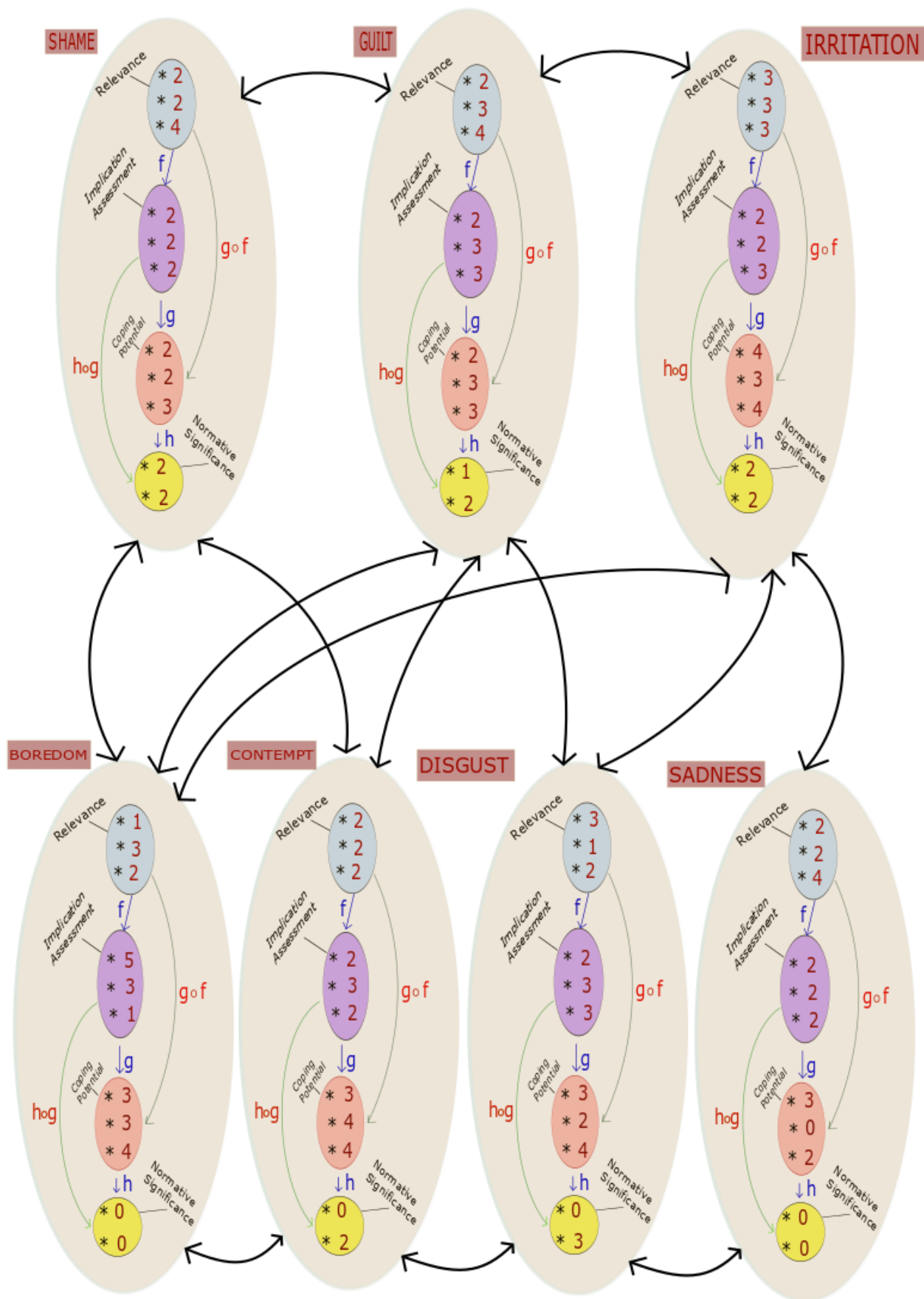


Figure 5.19: Relation between categories (Emotions) in the Depression

Figure 5.20 shows the last group develop with the algorithm *kmeans*. This group can be named as *Positive emotions* group.

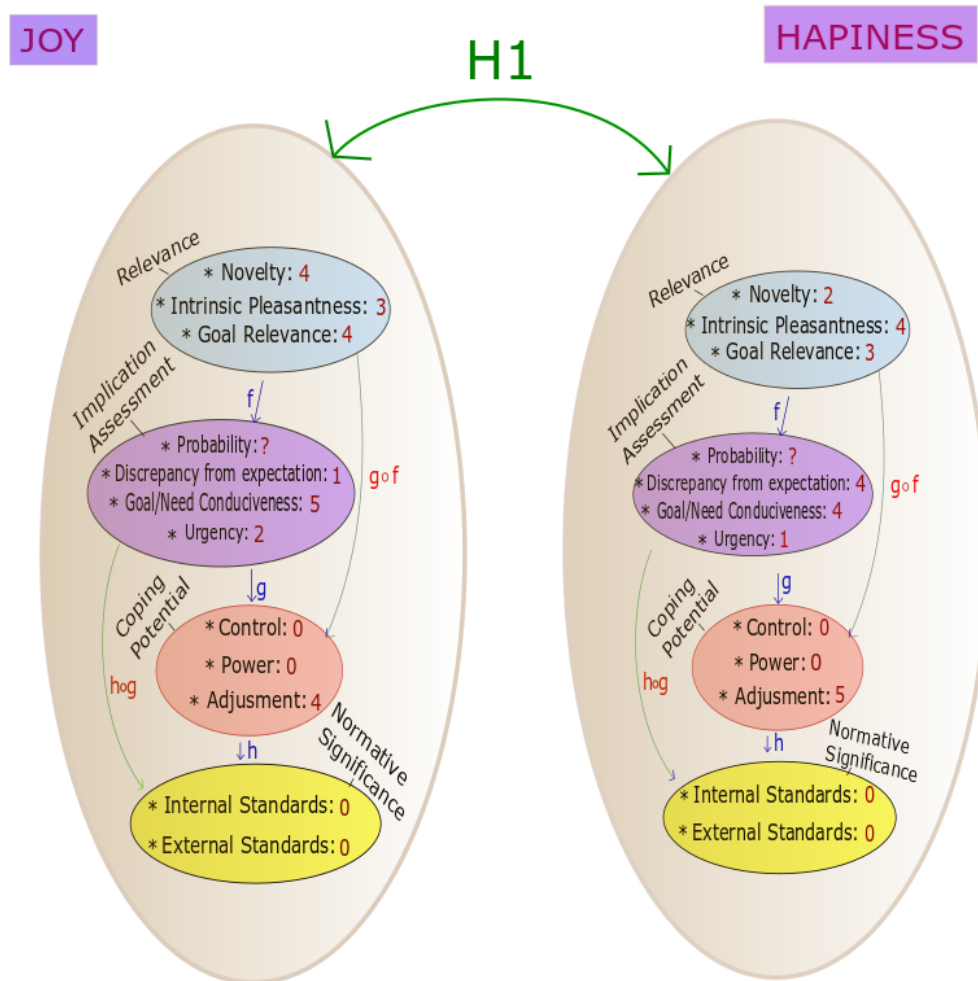


Figure 5.20: Relation between categories (Emotions) in the Positive emotions group

5.2 Proposal of OCC model as Categories

To do a computational model of human emotions is to think emotions as a system, as Scherer et al. (2001) says: “Human emotions is an organize set and it is possible to know what is its

process. In this case emotions are thinking that have an appraisal processes”.

In this way, an emotion in a human emerges if there is a situation that is evaluated in an appraisal process and then come up the emotion as a response to the situation (Scherer et al., 2001). There is a variety of appraisal theories of emotions, but all of them have in common that think emotions as a system. A kind of theories considers emotions in a discrete way, that is, there is no an infinite quantity (continuum) of emotions, but discrete emotions, each emotion perfectly distinguishable of other emotion (Scherer et al., 2001). It is the case of OCC, which emotions are consider in a discrete way.

Those theories are thinking as a system, then, it is possible to do a mathematical model, as (Adam et al., 2009), that is a mathematical formalization of OCC, and a computational model of them, as the implementations of some appraisal theories and showed in Figure 2.3. There is an example of this kind of models (a mathematical model). Hartmann et al. (2012) use Category Theory, in which, each emotion is a category and emotions are related with functors (functions with certain properties). It gives a mathematical formality to the psychological theory. One of the advantage is that it is possible to find contradictions in the theory, as (Adam et al., 2009), where the mathematical formalization of OCC has not contradictions in its structure. Another advantage is to find a computational model and to do an optimization in the model (Hartmann et al., 2012), and it is possible to test the psychological theory, as in the case of OCC theory (Ortony et al., 1990).

With this view, in this section is explained a view of OCC in CT, that is, in a mathematical way with a future propose of a computational implementation.

Using previews definitions of a category, Trujillo et al. (2018) propose an idea for OCC theory as categories. To model OCC theory with CT is used the general structure of OCC with respect to its variables.

It is possible to represent each group of type of emotions with a graph, that is each group of type of emotions is diferenciated by a unique graph. For example, Figure 5.21 shows a *graph* representing one set of *type of emotions*, in this case is the group *FORTUNE-OF-OTHERS* (Trujillo et al., 2018). The construction of the graph is based on the general structure of OCC showed in Figure 2.2.

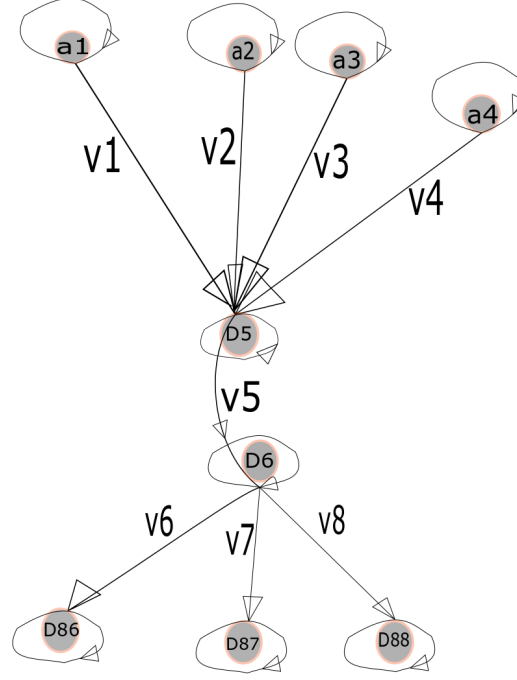


Figure 5.21: Type of emotion as a category (Trujillo et al., 2018)

In Figure 5.21, gray dots represent the global and local variables, these are:

- *Global variables* $a1, a2, a3, a4$ are: Sense of Reality, Proximity, Unexpectedness, Arousal.
- *Sets* $D5, D6, D86, D87, D88$ are defined to represent *Local Variables*: .

Now:

$$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 \quad (5.5)$$

Where:

$v_1, \dots, v_8$ represents arcs of the graph (in this case functions).

After that, it is necessary to define the set that is going to be used in the definition.

Denote $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $G = G_1 \cup G_2$, where $G_1 = \{a_1, a_2, a_3, a_4\}$ and $G_2 = \{a_5, a_6, a_7, a_8\}$. G_1 represents the values of global variables in red color in Figure ?? and G_2 represents the values of the rest of variables (local variables in blue color in Figure ??) that affects the set of emotion types **fortunes-of-others**, as it was explain in the above section.

To simplify the notation, let $i \in B$ for the rest of definitions and properties

$D = \{x \in \mathbb{R}^+\}$ be the set of possible values of the variables, that is: $a_i \in D$:

$$D_5 = \{\underbrace{(0, \dots, 0, a_i, 0, \dots, 0)}_{i\text{-position}} \in \prod_{j=1}^5 D : a_i \in G_1 \wedge i \leq 4\} \quad (5.6)$$

Elements of D_5 always have the fifth coordinate with zero and here some examples to understand the set $D_5 : (a_1, 0, 0, 0, 0), (0, a_2, 0, 0, 0), (0, 0, a_3, 0, 0)$. D_6 has a similar definition of D_5 with the difference that in the fifth coordinate has always the value a_5 , sixth coordinate is always zero and $D_6 \subseteq \prod_{j=1}^6 D$.

$$D_6 = \left\{ \underbrace{(0, \dots, 0, a_i, 0, \dots, a_5, 0)}_{i\text{-position}} \in \prod_{j=1}^6 D : a_i \in G_1 \wedge i \leq 4 \right\} \quad (5.7)$$

Examples of D_6 set are: $(0, 0, 0, 0, a_5, 0), (a_1, 0, 0, 0, a_5, 0), (0, 0, a_3, 0, a_5, 0)$. D_{86} has a similar definition to the D_6 set conserving the a_5 values in the fifth coordinate, D_{86} changes in those properties: it has in the sixth coordinate the value a_6 , seventh and eighth coordinates are always zero and $D_{86} \subseteq \prod_{j=1}^8 D$:

$$D_{86} = \left\{ \underbrace{(0, \dots, 0, a_i, 0, \dots, a_5, a_6, 0, 0)}_{i\text{-position}} \in \prod_{j=1}^8 D : a_i \in G_1 \wedge i \leq 4 \right\} \quad (5.8)$$

Examples of D_{86} set are: $(0, a_2, 0, 0, a_5, a_6, 0, 0), (0, 0, 0, a_4, a_5, a_6, 0, 0), (0, 0, a_3, 0, a_5, a_6, 0, 0)$. Sets D_{87} and D_{88} are defined in the same way as D_{86} set, they change values in the seventh and eighth coordinates:

$$D_{87} = \left\{ \underbrace{(0, \dots, 0, a_i, 0, \dots, a_5, 0, a_7, 0)}_{i\text{-position}} \in \prod_{j=1}^8 D : a_i \in G_1 \wedge i \leq 4 \right\} \quad (5.9)$$

$$D_{88} = \left\{ \underbrace{(0, \dots, 0, a_i, 0, \dots, a_5, 0, 0, a_8)}_{i\text{-position}} \in \prod_{j=1}^8 D : a_i \in G_1 \wedge i \leq 4 \right\} \quad (5.10)$$

Examples of sets D_{87} are: $(a_1, 0, 0, 0, a_5, 0, a_7, 0), (0, 0, a_3, 0, a_5, 0, a_7, 0), (0, a_2, 0, 0, a_5, 0, a_7, 0)$. Examples of sets D_{88} are $(a_1, 0, 0, 0, a_5, 0, 0, a_8), (0, a_2, 0, 0, a_5, 0, 0, a_8), (0, 0, 0, a_4, a_5, 0, 0, a_8)$.

After this, it continues with the graph notion. Where:

$O = G_1 \cup D_5 \cup D_6 \cup D_{86} \cup D_{87} \cup D_{88}$ and

$A = \{vi : i \in B\} \cup I$ ($I = \{id_c : id_c : c \rightarrow c \text{ identity arrow}\}$)

See Equation 2.4, where O is the set of objects and A is the set of arrows of the graph.

Next step is to define functions *Domain* and *Codomain* (see Equations 2.1 and 2.2).

Dom: $A \rightarrow O$

$$\text{Dom}(vi) = \begin{cases} a_i, & \text{if } i \leq 4 \\ D_5, & \text{if } i = 5 \\ D_6, & \text{if } 6 \leq i \leq 8 \end{cases} \quad (5.11)$$

$$\text{Cod}(vi) = \begin{cases} \text{Cod: } A \rightarrow O \\ D_5, & \text{if } i \leq 4 \\ D_6, & \text{if } i = 5 \\ D_{86}, & \text{if } i = 6 \\ D_{87}, & \text{if } i = 7 \\ D_{88}, & \text{if } i = 8 \end{cases} \quad (5.12)$$

The next step is to construct the set $A x_O A$ (see Equation 2.7).

$$Ax_O A = \{\langle vi, v5 \rangle : i \leq 4\} \cup \{\langle v5, vi \rangle : 6 \leq i \leq 8\} \quad (5.13)$$

The construction of the set $A x_O A$ is possible because of the definition of object relations in Equations 2.1 and 2.2, as well as, the constructions of the Domain and Codomain given in Equation 2.3.

According to definition of category, it is a graph with two additional functions and two additional axioms (see Equations 2.4 and 2.7).

Function identity

$$id : O = \{G_1 \cup D_5 \cup D_6 \cup D_{86} \cup D_{87} \cup D_{88}\} \rightarrow A \quad (5.14)$$

$$id(x) = id_x \quad (5.15)$$

Where $id_x : x \rightarrow x$ is the identity arrow.

In the case of $x \in O$, so, $x \in G_1$ or $x \in D_5$ or $x \in D_6$ or $x \in D_{86}$ or $x \in D_{87}$ or $x \in D_{88}$, but in each case goes to the identity arrow.

With respect to *Composition function*:

$$Ax_O A \xrightarrow{\circ} A \quad (5.16)$$

$$\langle vi, v5 \rangle \longrightarrow v5 \circ vi \quad \text{if } i \leq 4 \quad (5.17)$$

$$\langle v5, vi \rangle \longrightarrow vi \circ v5 \quad \text{if } 6 \leq i \leq 8 \quad (5.18)$$

Composition and associativity laws are proven in the following way:

For: $1 \leq i \leq 4$ and $6 \leq j \leq 8$

Composition

$$v_5 \circ v_i(a_i) = v_5(\underbrace{(\dots, a_i, \dots)}_{i\text{-position}}) \quad (5.19)$$

$$= (\dots, a_i, \dots, \underbrace{a_5}_{fifth\text{-position}}, 0) \quad (5.20)$$

$$v_j \circ v_5(\dots, a_i, \dots) = v_j(\dots, a_i, \dots, a_5, 0) \quad (5.21)$$

$$= (\dots, a_i, \dots, a_5, \dots, \underbrace{a_j}_{j\text{-position}}, \dots) \quad (5.22)$$

Associativity

To proof: $v_j \circ (v_5 \circ v_i) = (v_j \circ v_5) \circ v_i$.

$$v_j \circ (v_5 \circ v_i)(a_i) = v_j \circ v_5(\dots, a_i, \dots) \quad (5.23)$$

$$= v_j(\dots, a_i, \dots, a_5, 0) \quad (5.24)$$

$$= (\dots, a_i, \dots, a_5, \dots, a_j, \dots) \quad (5.25)$$

$$v_j \circ (v_5 \circ v_i) = (\dots, a_i, \dots, a_5, \dots, a_j, \dots) \quad (5.26)$$

$$(v_j \circ v_5) \circ v_i(a_i) = (v_j \circ v_5)(\dots, a_i, \dots) \quad (5.27)$$

$$= v_j \circ v_5(\dots, a_i, \dots) \quad (5.28)$$

$$= v_j(\dots, a_i, \dots, a_5, 0) \quad (5.29)$$

$$= (\dots, a_i, \dots, a_5, \dots, a_j, \dots) \quad (5.30)$$

With the above analysis it is proven *composition* and *associativity*.

With respect to *Unit law* the composition of functions is proven in the following way:

For each function v_k with $1 \leq k \leq 8$ it is constructed the following function:

$$1_{dom v_i} : dom v_i \rightarrow dom v_i \quad (5.31)$$

Where $dom v_i$ is the domain of the arrow v_i .

So, *Unit law* is guaranteed in the following way:

For $v_k : dom v_k \rightarrow cod v_k$ and $v_r : cod v_k \subseteq dom v_r \rightarrow cod v_r$ where $1 \leq k, r \leq 8$.

$$1_{domv_r} \circ v_k = v_k \quad (5.32)$$

$$v_r \circ 1_{domv_r} = v_r \quad (5.33)$$

5.3 Comparing Scherer's theory of emotions with OCC theory of emotion

The propose of a semantic study of emotions words is to establish a semantic connection between theories (OCC and Scherer).

In Ortony et al. (1987), the authors have propose a structure of the emotion lexicon to OCC. In Clore et al. (1987), the authors made an empirical study to verified the structure of the emotion lexicon mentioned before.

In Scherer's studies about emotion words, the focus is constructed from empirical studies and these studies explain that emotions words responds to a fixed pattern of the appraisals, i.e., if the studies presents a continue pattern of appraisal (same intensity, for example) then it is associated a word emotion to this pattern of appraisal (Fontaine et al., 2013).

This section, a first organization to links variables from OCC with checks from SECs (Scherer's theory) to establish a relationship between theories. This "link" is one of the biggest problems of Appraisal theories.

According to Scherer (1993):

"The lack of consensus on the number and types of appraisal criteria".

And Marsella et al. (2010) reinforce this idea:

"Emotions are a central aspect of everyday life and people have strong intuitions about them. As a consequence, the terms used in emotion research (appraisal, emotion, mood, affect, feeling) have commonsense interpretations that can differ considerably from their technical definition within the context of a particular emotion theory or computational model (Russell 2003). This ambiguity is confounded by the fact that there are fundamental disputes within psychological and neuroscience research on emotion over the meaning and centrality of these core concepts".

There is not a consensus about what is the optimal number and types of appraisals needed in an apraisal theory even so, there is not a consensus about what is an *Emotion*. In this way, the proposal research is to mitigate misunderstanding between emotion theories using category theory, semantic approach between emotion theories.

It is showed a first approximation of this future research. The idea is to establish an equivalence between variables in OCC and *Checks* in Scherer Appraisal's. In Table 2, the first variable in OCC that it is taken as an equivalent to the Check *Familiarity* in SECs is the variable *Familiarity* in OCC, because *Familiarity* in both theories is related with the bond of the subject experience the emotion with the event in Scherer (Scherer et al., 2001) and with the object in OCC (Ortony et al., 1990).

Another Check Scherer's is *Control Check* from the appraisal *Control Potential*, this check is about a situation that is possible to be controlled or not by the subject. For that reason, it is possible to establish a relation with the variable *Sense of reality* from OCC, where this variable determine the grade of reality of a situation, that is, if the situation has a sense of reality high then the experience of the emotion is high. It is possible to observe this relation in Table 2.

Next check is *Power Check*, if control is possible, this check evaluates resources of the subject to exert control of the situation. For that reason, it is possible to have a relation with the variable *Effort* from OCC, because *Power check* exert an *effort* in the subject to control the situation, to group the resources. *Effort* variable in OCC is related to the subject actions to achieve some goal, that is, to group resources to achieve this goal, if the effort is high the intensity of the emotion is high (Ortony et al., 1990).

Appraisal *Normative Significance Evaluation* in Scherer's is related to morality, to the action of others with respect to the person experience the emotion (Scherer, 1988). In OCC, morality and emotions provoked by the actions of others is related to *Attribution emotion*, where it is related to three variables: *Blameworthiness/Praiseworthiness*, *Strength of unit* and *Expectation deviation*. *Blameworthiness/Praiseworthiness* refers to the blame or praise of actions do it by the subject experienced the emotion. *Strength of unit* variable is related to the association of the subject experience the emotion with some person or group (action of others). For example: to feel pride of the achieve of some beloved person (Ortony et al., 1990). *Expectation deviation* is related to deviation expectation of the subject experienced the emotion has about some person or a person in some role. For example: a person can admire more a person that run a marathon without doing exercise than a person that do exercise all the days. For that reason is related to *Internal Standards Check* the variable *Expectation deviation* and related to *External Standards Check*, the variable *Strength of unit*. It is possible to establish another relationship between them, that is *Internal Standards Check* with *Strength of unit* and *External Standards Check* with *Expectation deviation*. The reason of these possibilities is that it is possible to establish the relation between those checks and variables according to their definitions.

Table 2 presents these relationships between appraisal theories. Words in red color means that are the above relation provided. Table 2 was modified from a table taken from (Scherer, 1993). In the table from (Scherer, 1993), there was not words in red color showed in Table 2:

Scherer	Ortony/Clore/Collins
Novelty	
Suddenness	
Familiarity	Familiarity
Predictability	Unexpectedness
Intrinsic Pleasantness	Appealingness
Goal significance	
Concern relevance	
Outcome probability	Likelihood
Expectation	Prospect realization
Conduciveness	Desirability
Urgency	
Coping potential	
Cause: Agent	Agency
Cause: Motive	Sense of reality
Control	Effort
Adjustment	
Compatibility Standards	Praiseworthiness
External	Strength of unit / Expectation deviation
Internal	Strength of unit / Expectation deviation

Table 2: Table comparing properties of Appraisal theories (with a computational implementation) - Making new connections between *Checks* in Scherer’s and variables in OCC.

One of the propose to relate theories of emotions through a semantic and in a categorical way has the intention to be applied in computational models of emotions, for that reason it is explained a context of computational models of emotion in next subsection.

6 CONCLUSIONS AND FUTURE WORKS

This work shows the necessity to provide bridges between theories of emotions to avoid confusion in theoretical concepts of emotions, to facilitate computational implementations and to improve emotion theories.

To achieve that, it was explained a little about history of appraisal theories, describing some of the appraisal theories, commenting about the computational models of emotions, development an study of checks using PCA, Corelation Pearons coefficient, k-means and CT.

In this research, it was proposed a model for OCC and Scherer's theory based in CT. To achieve that, it was study the structure of these theories, study an empiricial research developed by Scherer to use the data from the research and to provide connections between emotions of Scherer's theory and to provide connections between emotions from OCC. In Scherer's these connections was provided by PCA, that shows variance in a set of vectors, in this cases vectors of emotions given by the data. With respect to OCC, it was observed the relation between variables and Emotions as a graph, so each type of emotion is associated to a Graph and similar type of emotions has the same graph changing in intensity of the variables.

6.1 Future Works

OCC and Scherer's theories are Appraisal theories have intrinsic properties. OCC is a cognitive and semantic approximation to explain the process of emotions with a concept of *type of emotion* to refer to a set of 22 sets of emotions, in which each set has similar emotion labels. OCC theory is one of the theories that has more computational implementations, perhaps by its theoretical coherence, but it is a theory that it is not tested in empirical studies, as Scherer's theory.

Scherer's theory explains the inner process of appraisal (OCC does not do that), Scherer's theory consider emotions in a dimensional spaces, that is, there is an infinite quantity of emotions. Figure 6.1 shows some diference of these theories.

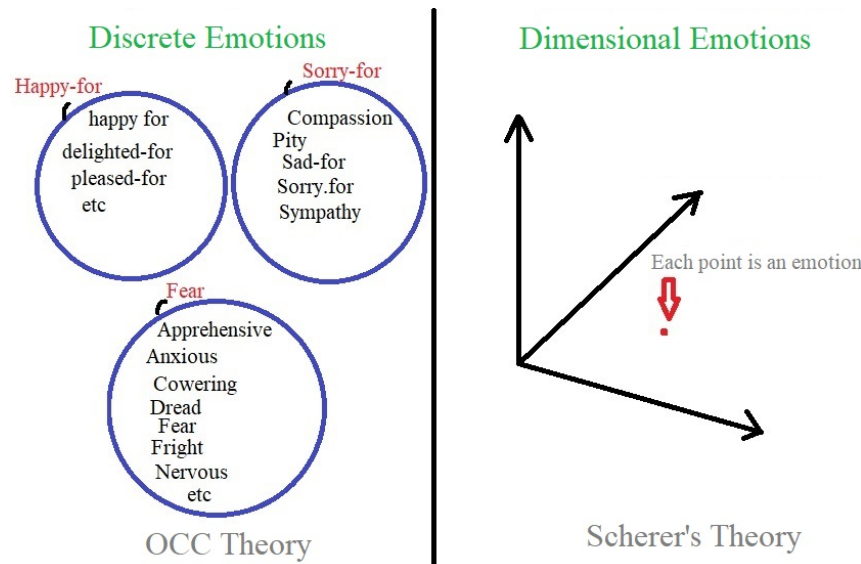


Figure 6.1: Discrete emotions *vs* dimensional emotions

Scherer mentions in (Scherer, 1993) and others authors as (Marsella et al., 2010), affirm that it is necessary to establish comparisons between emotion theories to mitigate the confusion in theoretical concepts and process of emotions elicitation, also, to improve emotion theories. With this propose, future works is to establish a comparison between these theories using category theory and a linguistic approximation.

As a first tentative, it is provided a way to compare these theories based of previews Scherer's studies taking in account some restrictions:

1. It is contemplated a finite number of emotions in the dimensional space of emotions in Scherer theory;
2. The relation provided between these theories is a semantic relation between variables in OCC and Checks in Scherer's;
3. Data is taken from empirical studies developed by Scherer to test his theory in the prediction of emotions. This data is used to make the semantic comparison between theories.

With respect to the first restriction, it is necessary to establish a finite number of emotions to do the comparison between theories because Scherer's studies testing his theory use a finite number of emotions (Scherer, 1993) (Meuleman and Scherer, 2013).

With respect to the second restriction, it is conceived in that way because theories are so different that the unique approximation it a semantic way.

With respect to the third restriction it is considered in this manner because there are only empirical data from Scherer's theory.

Once, it is established the restrictions, next step is to explain the semantic approximation between theories. In table 2 is exposed some semantic similarities between OCC and Scherer's

theory, but it is necessary to improve these similarities with the future propose of to establish a better relation of the theories using category theory.

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