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Dissertação de Mestrado

Planejamento de Trajetória de um Veículo Híbrido Aéreo-aquático Baseado em Heurísticas para Eficiência Energética

Pedro Miranda Pinheiro

Dissertação de Mestrado apresentada ao Programa de Pós-Graduação em Computação da Universidade Federal do Rio Grande - FURG, como requisito parcial para a obtenção do grau de Mestre em Engenharia de Computação

Orientador: Prof. Dr. Paulo Lilles Jorge Drews-Jr.
Co-orientador: Prof. Dr. Armando Alves Neto

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Master of Science Dissertation

Trajectory Planning of an Aerial-Underwater Hybrid Vehicle Based on Heuristics for Energy Efficiency

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DISSERTAÇÃO DE MESTRADO

**Planejamento de Trajetória de um Veículo Híbrido Aéreo-aquático
Baseado em Heurísticas para Eficiência Energética**

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RESUMO

PINHEIRO, Pedro Miranda. **Planejamento de Trajetória de um Veículo Híbrido Aéreo-aquático Baseado em Heurísticas para Eficiência Energética.** 2023. 67 f. Dissertação (Mestrado) – Programa de Pós-Graduação em Computação. Universidade Federal do Rio Grande - FURG, Rio Grande.

Este trabalho aborda o planejamento de trajetória de um veículo híbrido aéreo-aquático não-tripulado chamado *Hydrone*, que está sendo desenvolvido no Grupo de Robótica e Automação Inteligente (NAUTEC) da Universidade Federal do Rio Grande (FURG). Este trabalho é baseado em uma técnica clássica para o planejamento de trajetória de veículos autônomos, árvores de exploração rápida, conhecida pela sigla em inglês, RRT (*Rapidly-exploring Random Tree*). Ainda, para o aumento de rendimento energético do veículo, são propostas duas heurísticas. A primeira destas envia a expansão da árvore para o domínio de menor custo de navegação, e a segunda atribui custos estimados para os nós da árvore e escolhe os caminhos mais baratos. Este trabalho demonstra as principais características e potenciais de aplicação destes métodos para a navegação de veículos híbridos. Para avaliação das técnicas, foram realizados experimentos simulados em 135 diferentes cenários utilizando cada um dos três algoritmos comparados, o algoritmo sem heurísticas e os dois algoritmos com as heurísticas propostas. Também é apresentada a análise comparativa de suas performances. Foi identificado nestes resultados, que a utilização de heurísticas pode contribuir de forma significativa para a redução do consumo de energia, e até mesmo para o aumento de velocidade média nas missões realizadas por estes veículos. Dentre os principais resultados, foi identificado que em 95% dos cenários, um dos algoritmos que utilizam as heurísticas propostas foi o responsável pelo menor consumo de energia. Por fim, as descobertas deste trabalho são resumidas e são identificadas oportunidades de trabalhos futuros.

Palavras-chave: Robótica móvel, veículo híbrido, planejamento de trajetória.

RESUMO

PINHEIRO, Pedro Miranda. **Trajectory Planning of an Aerial-Underwater Hybrid Vehicle Based on Heuristics for Energy Efficiency**. 2023. 67 f. Dissertação (Mestrado) – Programa de Pós-Graduação em Computação. Universidade Federal do Rio Grande - FURG, Rio Grande.

This work addresses the trajectory planning of an unmanned hybrid aerial-underwater vehicle called *Hydrone*, which is being developed by the Intelligent Robotics and Automation Group (NAUTEC) of the Federal University of Rio Grande (FURG). This work is based on a classical technique for trajectory planning of autonomous vehicles, Rapidly-exploring Random Trees (RRT). In addition, two heuristics are proposed to increase the energy efficiency of the vehicle. The first of these heuristics consists of biasing the tree expansion towards the domain with the lowest navigation cost, and the second assigns estimated costs to nodes in the tree and chooses the cheapest paths. This work demonstrates the main characteristics and application potentials of these methods for hybrid vehicle navigation. To evaluate these techniques, simulated experiments were performed in 135 different scenarios using each of the three compared algorithms, the algorithm without heuristics and the two algorithms with the proposed heuristics. A comparative analysis of their performances is also presented. It was identified in these results, that the use of heuristics can contribute significantly to the reduction of energy consumption, and even increase the average speed in the missions performed by these vehicles. Among the main results, it was identified that in 95% of the scenarios, one of the algorithms using the proposed heuristics was responsible for the lowest energy consumption. Finally, in the last part of this work, the findings are summarized and opportunities for future work are identified.

Palavras-chave: Mobile Robotics, hybrid vehicle, motion planning.

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LIST OF ABBREVIATIONS AND ACRONYMS

HUAUV	Hybrid Unmanned Aerial Underwater Vehicle
UAV	Unmanned Aerial Vehicle
UUV	Unmanned Underwater Vehicle
AUV	Autonomous Underwater Vehicle
ROV	Remotely Operated Vehicle
FURG	Universidade Federal do Rio Grande
NAUTEC	Research Group on Automation and Intelligent Robotics
ROI	Return over investment
PRM	Probabilistic Roadmap
RRT	Rapidly-exploring Random Trees
PID	Proportional–Integral–Derivative
SLAM	Simultaneous Localization and Mapping
DDPG	Deep Deterministic Policy Gradient
EKF	Extended Kalman Filter
UKF	Unscented Kalman Filter

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1 INTRODUCTION

1.1 Hybrid Unmanned Aerial Underwater Vehicles

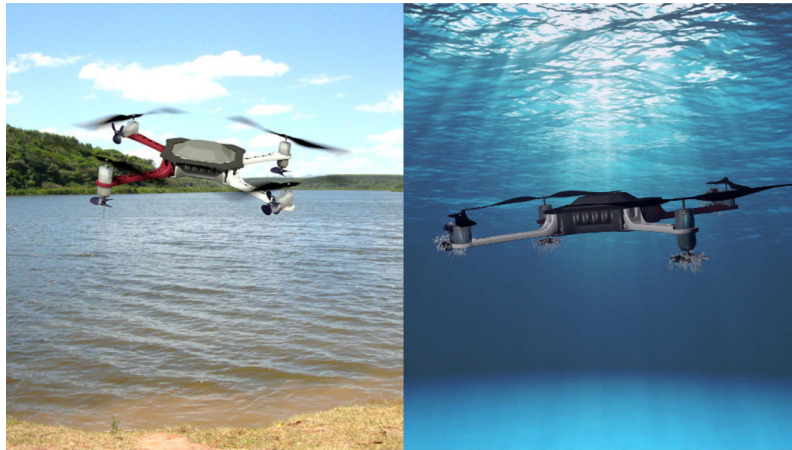
Hybrid Unmanned Aerial Underwater Vehicles (HUAUVs) are a class of mobile robots that combine the characteristics of both unmanned aerial vehicles (UAVs) and unmanned underwater vehicles (UUVs). The idea of having vehicles that can navigate in both media is not new (DREWS; NETO; CAMPOS, 2009). However, only in the last decade, with modern hardware and software, has the topic achieved a more feasible scale for research and development.

By having the ability to navigate in two different media, HUAUVs are capable of obtaining a complete perception of environments close to water, such as riverbanks, ports, oil platforms, and even flooded caverns. The more complete perception enables activities such as full inspection of partly submerged structures, environmental monitoring, and assessment, and search and rescue without the need for human assistance to switch between environments.

However, navigating in both media demands a platform that can adapt to two densities that are almost three orders of magnitude apart. It also has to be lightweight and waterproof. These challenges are not trivial and are being studied by research groups in many universities worldwide (SIDALL; KOVAC, 2016; WEISLER et al., 2017; ALZU'BI; MANSOUR; RAWASHDEH, 2018; MA; FENG; YANG, 2018; LU et al., 2019; HORN et al., 2020; CHEN et al., 2020).

At Universidade Federal do Rio Grande (FURG), the Research Group on Automation and Intelligent Robotics (NAUTEC) has been working on a proof-of-concept HUAUV called Hydrone (Figure 1). The prototype is still being developed. However, significant contributions have already been made in the modeling and control of HUAUVs (ROSA et al., 2018; HORN et al., 2020; HORN et al., 2019; GRANDO et al., 2020; GRANDO et al., 2021; AOKI et al., 2022b; AOKI et al., 2022a; AOKI et al., 2021; PEDROSO et al., 2022; GRANDO et al., 2022a; GRANDO et al., 2022b). This work is based on the Hydrone platform described by Horn et al. (2020).

Figure 1: Hydron concept (HORN et al., 2020)



1.2 Motivation

The research field on HUAUVs has yet to cover two important topics: robot planning systems and energy efficiency. Although functional prototyping and control systems are more urgent, planning systems and energy efficiency are crucial for the wider adoption of these systems. With well-developed planning systems, complex missions can be accomplished, and with more energy-efficient systems, mission length can be increased when using batteries.

1.2.1 Hybrid aerial-underwater vehicles trajectory planning

HUAUVs have unique characteristics that distinguish them significantly from other robotic systems. These include the ability to transition between different media, such as air and water, and to navigate in fluids with distinct properties. These characteristics have a direct impact on the vehicle's trajectory planning. Since trajectory planning defines the best way for the vehicle to move in free space, these characteristics raise important questions to be studied.

For example, moving in different fluids generates different dynamics, which may or may not prevent the robot from moving safely. Also, if the planner's goal is to optimize a specific parameter, such as energy or time, the fact that there are two options of media in which to move, as well as a cost associated with the transition between them, also raises new problems to be studied. Airborne travel, for example, could possibly allow faster flight. Still, the weight to be supported by the propulsion system would be greater, and the operation dynamics are different than underwater travel.

1.2.2 Energy efficiency

As a class of mobile robots that mainly operate in two fluids (air and water), HUAUVs need constant actuation even to maintain their position. This represents a higher energy consumption when compared to other types of mobile robots, e.g., terrestrial robots and

water surface robots (in calm water and wind conditions).

Lower energy consumption affects the operation of these systems in two ways. The first is the increase in the operating time of these systems, which would allow, for example, for covering greater distances at the same average speed. The second is the reduction in energy costs, which can define the adoption or not of these platforms in projects that use these robots extensively. Although these are simple questions that separately may seem insignificant, their combination could greatly facilitate the application of these systems in real missions in industrial or logistics environments.

Therefore, it's important to understand the behavior of these platforms and identify the best strategies to lower their energy demand. Doing so can help increase their operation time, and availability, and ultimately improve their overall productivity.

1.2.3 Industrial application and offshore scenario

Robotic systems are becoming more and more affordable due to new, cost-effective solutions in actuation and sensing. However, robots that are as versatile and reliable as humans are still expensive for most companies. Nevertheless, high-risk and time-consuming human activities can sometimes be performed as well as, or even more effectively, by robotic platforms.

The offshore oil and gas industry has many challenging environments that pose a high risk to human workers. This includes activities at high altitudes, exposure to toxic gases, and the risk of explosions (ASAD et al., 2019). Therefore, one of the main concerns of modern oil and gas companies is to find ways to remove the human presence from such high-risk activities.

Furthermore, many countries around the world have offshore fossil fuel reserves, including Mexico, the United States, and Brazil (SANDREA; SANDREA, 2007). As a result, there are a significant number of partially submerged structures that require inspection and constant monitoring to meet global health, safety, and environmental (HSE) and maritime standards.

These situations present an ideal opportunity for the use of HUAUVs. Industries operating in hybrid environments and performing high-cost and high-risk activities could greatly benefit from the use of these systems. By employing HUAUVs, companies can improve worker safety and reduce the risk of costly accidents or incidents, while also ensuring that their equipment meets the necessary inspection and monitoring requirements.

In this way, enabling these hybrid platforms to carry out complex missions efficiently may enable their employment in their first industrial activities, thus creating a research, development, and application ecosystem that would foster the consolidation of this field of Robotics.

1.3 Problem

One of the key problems with navigating through industrial facilities is planning a trajectory that is both efficient and safe. An efficient trajectory achieves a goal with minimum consumption of resources, e.g., moving to a specific location or surveying a certain area in a minimum amount of energy. And a safe trajectory guarantees the physical integrity of the robot and everything/everyone around it, usually avoiding collisions and maintaining safe distances.

Adding to the industrial navigation problem, some industries (e.g., the oil and gas industry) also have partially submerged structures, and a vehicle that navigates in both environments (HUAUV) must be able to plan a hybrid trajectory. A hybrid trajectory includes environment transition. Thus, the system planner must be able to deal with the change in dynamics due to a significant media density difference. This problem is defined in Problem 1.

Problem 1. Let a HUAUV be modeled with distinct dynamics for aerial and aquatic operation, with their respective control laws. This vehicle navigates in an environment $\mathcal{X}$, with static obstacles. Let it also be assumed the existence of a transition zone around the water surface at $z = 0$, in the range $-\mu \leq z \leq \mu$, for $\mu \geq 0$. And finally, assume a cost function to be provided such that it relates the vehicle's actions to the energy consumption along a trajectory. Then, the main goal is to find a trajectory from the vehicle's starting point to a given goal ($\mathbf{r}_{\text{goal}}$), such that $\mathbf{x}(t) \in \mathcal{X}_{\text{free}}$ for all $t > 0$. The planner algorithm must also minimize energy consumption and allow for a smooth transition between environments.

1.4 Objective

To deal with the previously mentioned topics, the main objective of this work is to develop a planning algorithm capable of creating an efficient, feasible, and collision-free hybrid trajectory to be followed by the Hydrone vehicle.

1.4.1 Specific objectives

To achieve the main objective, the following tasks must be completed:

- Improvement of Hydrone mathematical model;
- Improvement of simulation and design of appropriate simulated scenario;
- Formulation and coding of trajectory planning algorithm;
- Evaluation and analysis of planning algorithm performance.

1.5 Contribution

This work addresses a gap in the literature on HUAUVs by providing a trajectory-planning method for multirotor-based HUAUVs. Additionally, another relevant topic is addressed: the energy efficiency of mobile hybrid aerial-underwater robots. These are not trivial subjects, as multi-domain robotics techniques are not yet consolidated and the energy efficiency problem has many associated trade-offs. However, these challenging topics could, for example, change the way robots participate in industrial activities.

Having an efficient trajectory planner for multirotor-based HUAUVs may enable future industrial applications. Since one of the main constraints associated with mobile robots is the limited energy capacity, the more efficient their performance, the larger their availability becomes. Thus, the return on investment (ROI) may become more attractive to industrial enterprises, leading to an increase in the range of economically viable applications for these vehicles.

This work presents two main contributions:

- The first one is a trajectory planning algorithm for a multirotor Hybrid Unmanned Aerial Underwater Vehicle;
- The second are two heuristics for energy-saving trajectory planning.

1.6 Publication

As already mentioned, the field of study of Robotics for HUAUVs is still quite preliminary, although it is growing rapidly. Thus, throughout the development of this work, studies were naturally carried out in areas complementary to trajectory planning, such as modeling, control and simulation. From these studies, some of the contributions include developing visual-based navigation systems, improving vehicle performance through new designs and control algorithms, studying the effects of media transition and ground effect, and prototyping HUAUVs with off-the-shelf components (GRANDO et al., 2020; HORN et al., 2020; AOKI et al., 2021; AOKI et al., 2022b; AOKI et al., 2021; AOKI et al., 2022a; PEDROSO et al., 2022). The work also explores the use of deep reinforcement learning for mapless navigation and obstacle avoidance in both aerial and underwater environments (GRANDO et al., 2021; GRANDO et al., 2022a; GRANDO et al., 2022b).

Part of this work dealt with the implementation of a trajectory planning algorithm based on RRTs, called Hybrid Closed-loop RRT. In this work by Pinheiro et al. (2022), an online trajectory planner for hybrid unmanned aerial-underwater vehicles, capable of collision avoidance against unknown obstacles and smooth transitions between aerial and aquatic media, was developed. The approach used the dynamic model of the Hydrone, but it is believed that it can be easily generalized to other types of hybrid platforms. Some simulated results in environments filled with obstacles, where the robot was commanded to

perform different media movements, demonstrated the applicability of the adopted strategy.

In the next section, some of the most significant contributions made to the HUAUV and mobile robot motion planning fields are introduced (Section 2); then, the vehicle model, the simulation, the motion planning algorithms, and the algorithm performance evaluation utilized in this work are presented (Section 3); furthermore, the results are presented and discussed (Section 4); and finally, a summary of the achievements of this work and the prospects for the dissertation are addressed (Section 5).

2 LITERATURE REVIEW

2.1 HUAUVs

The state of the art in HUAUVs is not as extensive as it is in other areas of mobile robotics, but the progress made is significant for such a young field. Most of the research was done after the 2010 decade, with authors focusing primarily on proposing concepts and modeling them. After some proposed models, the authors focused on both the control and manufacturing of some of these models. Another concern addressed by some authors was energy efficiency, a major topic for mobile robotics that represents another challenge for those involved in the HUAUV design, manufacture, and testing.

In the next subsections, the most significant state-of-the-art studies are presented and the key achievements and contributions of each one are discussed.

2.1.1 Multirotors

Multirotors are aerial vehicles that generally make use of pairs of propellers in radial patterns to overcome gravity and navigate. Navigation typically necessitates changing the vehicle's attitude, and controllers for both position and attitude are simpler than those used for other aircraft, such as fixed-wing airplanes, because the multirotor only needs to unbalance the force generated by each propeller to directly actuate in four degrees of freedom.

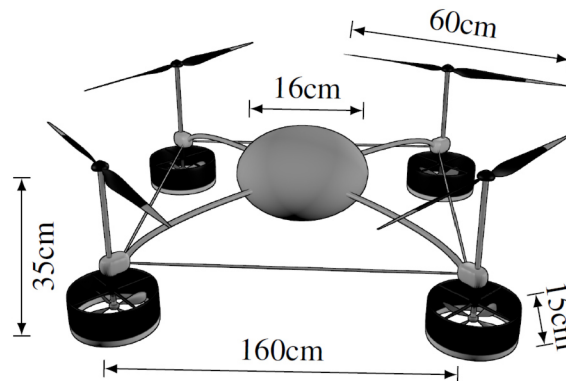
HUAUVs based on multirotor platforms follow the same principle, which enables them to perform missions that require hovering, vertical landing, and takeoff. Multirotors can perform such tasks with relative ease since their motors usually point upwards. Nevertheless, this characteristic also means that multirotors use more energy than other classes of HUAUVs.

Among many design choices, a particular one for the multirotor HUAUVs is choosing the underwater vertical actuator. And, as presented in this work, these systems can range from ballasts to aquatic-specific propellers.

A work by Drews, Neto and Campos (2014) presents the modeling and simulation of one of the first small-scale hybrid unmanned aerial underwater vehicles based on quadro-

tor platforms. The paper begins with a qualitative analysis of the many UAV and UUV platforms on several topics. As a result of the platform classification, the authors focused on a specific geometry for the HUAUV, shown in Figure 2. Subsequently, a model is defined for the chosen geometry and a proportional-derivative (PD) and nonlinear controllers are developed to stabilize the vehicle.

Figure 2: Concept geometry and design



Source: Drews, Neto and Campos (2014).

In the following works, the authors also produced an attitude control for an HUAUV (NETO et al., 2015), a comparative study on sigma-point Kalman filters for trajectory estimation (ROSA et al., 2018), a study on the configuration of propellers for multirotor HUAUVs (HORN et al., 2019), and a novel concept for HUAUVs focused on aquatic performance (HORN et al., 2020).

Following a similar workflow, Maia, Soni and Diez (2015) analyzes the key features of aerial and aquatic specialized animals, extracting the goals for the main features of a HUAUV prototype. After defining the goals, the seamless transition method is presented, where a double set of four propellers each provides the means for actuating during the transition. Then, the propulsion system is described, and its simulated performance is shown. Finally, a working prototype named Naviator is presented, as shown in Figure 3, together with the preliminary results, thus proving the feasibility of the proposed concept.

The Naviator was consolidated as a state-of-the-art prototype in the work by Maia, Mercado and Diez (2017), where the authors further detail the concept by presenting the dynamic model, control scheme, other simulated experiments, and real-time experiments. Additionally, Mercado, Maia and Diez (2019) have produced a more advanced control system by introducing the use of quaternions for the orientation representation and a non-linear feedback control loop for the attitude control. Also, Ravell, Maia and Diez (2018) have proposed a new strategy for the media transition.

Another multirotor-like HUAUV is presented in Alzu'bi, Mansour and Rawashdeh (2018), where the authors present a detailed design description of the vehicle concept, which includes four aerial-specialized propellers and a ballast system. Because the depth

Figure 3: Naviator Prototype



Source: Maia, Soni and Diez (2015)

control system distinguishes it from other HUAUVs, the authors detail the main design choices and constraints, such as the positioning of the centers of buoyancy (C_B) and gravity (C_G). By using the ballast system, the concept named Loon Copter (shown in Figure 4), can control the vertical axis position with very little energy consumption.

Figure 4: Loon Copter prototype



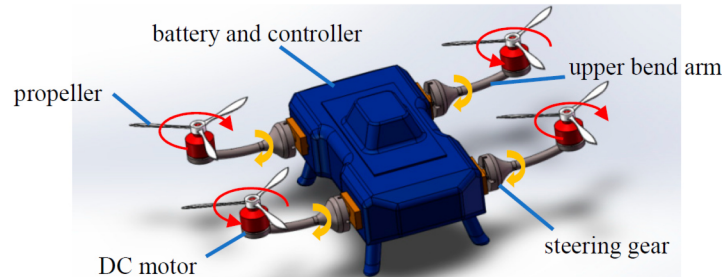
Source: Alzu'bi, Mansour and Rawashdeh (2018)

As presented in detail in a previous work (ALZU'BI et al., 2015), the authors also describe the process of evaluation of the HUAUV power plant and the selection of the motor-propeller pair. Furthermore, the authors describe the electronics and controls of the vehicle and its performance. And finally, the general performance of the HUAUV is presented, proving the concept and consolidating it as one of the state-of-the-art working prototypes.

In the work by Chen et al. (2019), a four-rotor vehicle with adjustable arms is proposed, as shown in Figure 5. By changing the rotor orientation, the vehicle produces a downward thrust to overcome its positive buoyancy. Also, the authors derive the model for the continuous dynamics of the concept according to Newton-Euler formalism. After presenting the dynamic model, the work demonstrates robust sliding mode switching

controllers and their application in simulation.

Figure 5: Proposed concept with adjustable arms



Source: Chen et al. (2019)

Tan and Chen (2019) present an aerial-aquatic multirotor with morphing capabilities (Figure 6) that was designed and built. The authors propose the use of four aerial propellers and a mechanism that allows them to be rotated along the axis of the respective arm. By using the same propellers for actuating in both media, the total mass would be lower and the vehicle would be fully actuated.

Figure 6: Rotating arms prototype

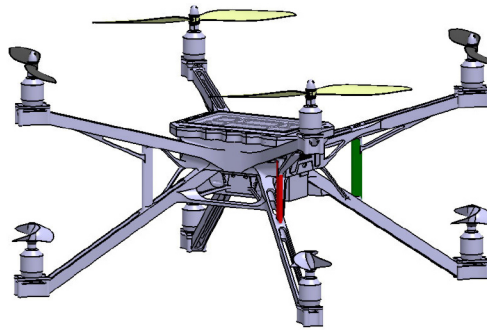


Source: Tan and Chen (2019)

Another work that addresses the control challenge is the one by Lu et al. (2019). In this paper, the authors adopt a generic model of HUAUV with four propellers pointing upwards and a central hull. Also, the dynamic model for the concept of the vehicle based on Newton-Euler formalism is presented. For the control, an adaptive dynamic surface control scheme is proposed, and the stability is proved. Finally, the authors validate the proposed control through a simulation of the vehicle.

Lastly, a concept was proposed by Chen et al. (2020), as shown in Figure 7. Similar to the concept proposed by Drews, Neto and Campos (2014), the vehicle has four aerial specialized propellers and four aquatic propellers. Using an incremental nonlinear dynamic inversion control, this work focuses on attitude and altitude control.

Figure 7: Eight arms proposed concept



Source: Chen et al. (2020)

2.1.2 Fixed-wing

Fixed-wing UAVs are similar to commercial airplanes and are a popular class of aerial robots. Their main applications are those that require long endurance, such as mapping large areas, surveillance, and monitoring. Differently from multirotors, they rely on only one motor to generate the required thrust, thus needing their wings to generate lift and their flaps to perform changes in their orientation.

However, fixed-wing HUAUVs present an even greater challenge than multirotors with respect to the media transition. That is due to the fact that they need momentum to produce the required lift for take-off. Reaching this stage requires overcoming large drag forces produced by the vehicle-water interface. Additionally, the same drag forces act during the landing in the water, meaning that reaching the water surface at high speed may damage and/or incapacitate the vehicle.

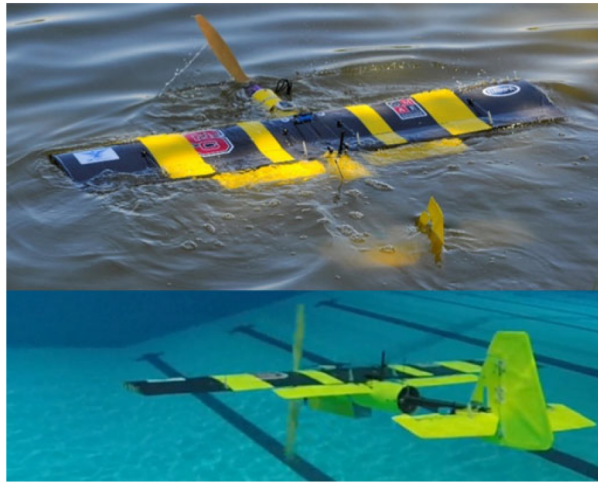
Therefore, fixed-wing HUAUV concepts and built prototypes available in the literature may not be as many as their multirotor counterparts, but they represent a significant part of the existing work in the HUAUV field of study.

The work by Weisler et al. (2017) presents one of the first fixed-wing HUAUV concepts and prototypes, shown in Figure 8. The vehicle has a 1.5m passively flooding wing and can perform multiple full-domain transitions. This means that the horizontally launched vehicle performs aerial flight, almost horizontal water ingress, underwater cruise, and vertical water egress.

In Weisler et al. (2017), the authors also provide a detailed description of design and manufacturing processes, hardware specifications, and vehicle performance. Additionally, in another work by Stewart et al. (2019), the authors further describe the passively draining wing and its dynamic modeling.

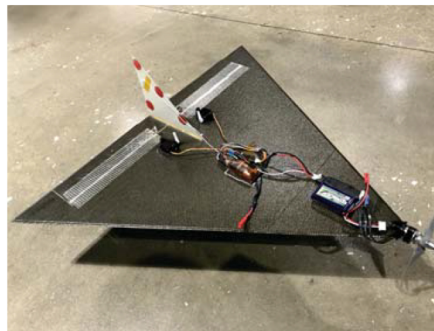
Another fixed-wing HUAUV concept was proposed by Moore, Fein and Setzler (2018). Different from the previous prototype, this concept has delta-wings, which provide a larger buoyancy assist during the water-air transition. Also, a proof-of-concept prototype was built (Figure 9) and tested in a remotely controlled application.

Figure 8: Fixed-wing prototype



Source: Weisler et al. (2017)

Figure 9: Delta-wing prototype



Source: Moore, Fein and Setzler (2018)

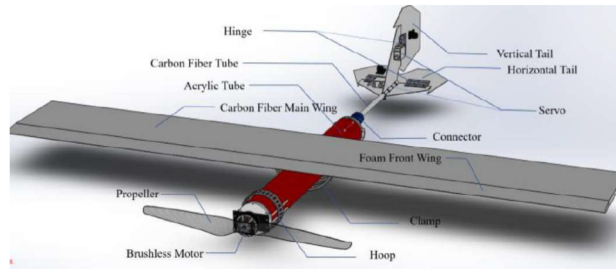
Both of the works mentioned above featured vehicles that cruised in the air and underwater in a similar manner. However, in the one by Wang et al. (2019), the authors propose a novel concept of fixed-wing HUAUV that combines a typical fixed-wing UAV with an underwater glider, shown in Figure 10.

2.1.3 Bio-inspired

In the work by Siddall, Ancel and Kovač (2017), one of the first concepts of bio-inspired HUAUVs is proposed, and a proof-of-concept prototype is built. The AquaMAV (Figure 11) uses a morphing structure to deploy and retract its wings. Its design is inspired by the behavior of aquatic birds (e.g., the northern gannet), which can perform plunge-diving.

Another feature of this robot design is the use of a jet thruster, developed by the authors and further explained by Siddall and Kovac (2016). The thruster is used to provide the necessary momentum for the vehicle to emerge and take-off from the water. Thus, allowing for planar motion, since a significant altitude is reached when exiting the wa-

Figure 10: Fixed-wing proposed concept



Source: Wang et al. (2019).

Figure 11: Bio-inspired prototype



Source: Siddall, Ancel and Kovač (2017)

ter, and the prototype does not rely *only* on planar motion since the propulsion system presented by Tan, Siddall and Kovac (2017) is also applied to the vehicle.

2.1.4 Considerations

As shown in Table 1, the state of the art of HUAUVs is mainly composed of multirotor concepts and prototypes, possibly due to their design simplicity, ease of manufacturing, and control. Additionally, the vertical take-off and landing (VTOL) capabilities play an important role in mission flexibility since multirotors can even be deployed from confined spaces and moving vehicles. Multirotors can also perform hovering flights, allowing for sensors and other systems to have a better perception of the surrounding space.

However, the lack of endurance of multirotors limits the reach of the robot and the feasibility of long-range missions, even if powered by large battery packs. This conceptual limitation derives from the pointing-upwards thrust vector, meaning that a large portion of the energy of the robot is applied solely to its lift.

Therefore, the second most common HUAUV class is the fixed-wing class. This class of vehicle generally has a higher top speed and better motion efficiency in long-range missions. Also, fixed-wing HUAUV aerodynamics are not trivial, but when properly designed, yield low drag forces and high lift.

The last discussed category is the bio-inspired HUAUVs, which are present in the state of the art. However, the limited range of applications and complex characteristics of prototypes are still significant for this class. Since bio-inspired prototypes tend to have

Table 1: State of the art proposed concepts on HUAUVs.

References	Research institution	HUAUV class
Drews, Neto and Campos (2014)	Universidade Federal do Rio Grande and Universidade Federal de Minas Gerais	Multicopter
Maia, Soni and Diez (2015)	Rutgers University	Multicopter
Alzu'bi, Mansour and Rawashdeh (2018)	Oakland University	Multicopter
Chen et al. (2019)	Dalian Maritime University and Dalian University of Technology	Multicopter
Tan and Chen (2019)	National University of Singapore and the Chinese University of Hong Kong	Multicopter
Chen et al. (2020)	Airforce Engineering University and Aviation University of Air Force	Multicopter
Weisler et al. (2017)	North Carolina State University and Teledyne Scientific and Imaging	Fixed-wing
Moore, Fein and Setzler (2018)	Johns Hopkins University and University of Maryland	Fixed-wing
Wang et al. (2019)	Shanghai Jiao Tong University	Fixed-wing
Siddall, Ancel and Kovač (2017)	Imperial College London	Bio-inspired

complex geometries and moving parts, they increase the overall complexity of an inherently complex problem, i.e., the design and manufacturing of HUAUVs. These and the limited array of manufacturing techniques available in the majority of research institutes may be the reason why bio-inspired HUAUVs are still a small portion of the state of art of HUAUVs.

Lastly, the HUAUV field of study is a challenging ecosystem that requires both creativity and a high level of technical knowledge and infrastructure. Many questions are still only partially or not at all addressed by state-of-the-art research, presenting an opportunity for research groups to achieve many more findings. Some of the topics that are yet to be well-covered on HUAUVs are transition control; energy efficiency; path planning and trade-offs; and industrial applications.

2.2 Path planning

2.2.1 Definition

Path planning can be interpreted as a geometric problem, i.e., finding a collision-free pathway between an initial and a final configuration (GASPARETTO et al., 2012). Planning for a static and known environment is simpler than for an unknown one with dynamic obstacles. However, these characteristics are only a few of many others that involve plan-

ning a feasible path for a mobile robot. Path planning algorithms search through a state space $\mathcal{X}$, and try to find a path in a free space $\mathcal{X}_{free}$ by avoiding obstacles. Among the classical path planning algorithms are the following: roadmap, cell decomposition, potential field, and probabilistic roadmap planner.

2.2.2 Algorithms

The roadmap methods capture the free space connectivity on a system of 1-dimensional curves in $\mathcal{X}_{free}$ space called the roadmap. The fitting of a graph to the space enables the path planning to take place by searching the roadmap for the proper path (GASPARETTO et al., 2012; GOERZEN; KONG; METTLER, 2010).

Cell decomposition approaches divide the space of free configurations into regions called cells. These are created by choosing heuristics that provide unique characteristics for the next phase. After the decomposition, a connectivity graph, where each vertex stands for a region, can be created to represent the adjacency of the cells. The problem is then reduced to finding a continuous free path from the initial configuration to the goal configuration in the graph (GASPARETTO et al., 2012; GOERZEN; KONG; METTLER, 2010).

The potential field methods introduce a powerful heuristic: a point/particle stands for the robot in the configuration space and moves according to the attractive and repulsive potential of the target and obstacles, respectively. This approach allows for a faster reaction to unexpected changes in the workspace, finding the most promising free path. However, it also produces a local minima trap that can completely stop the robot, and it requires the discretization of the $\mathcal{X}$ space into a grid (GASPARETTO et al., 2012).

The probabilistic roadmap (PRM) alternative applies probabilistic techniques to produce the roadmap. The major difference to the other approaches is that the PRM complexity is a function of the path complexity rather than a function of the environment complexity and size (GASPARETTO et al., 2015). Therefore, this approach admits a solution to problems of arbitrary complexity and dimension that is probabilistically convergent (GOERZEN; KONG; METTLER, 2010).

In conclusion, there are several path-planning algorithms and techniques that can be used for calculating a feasible path for a robot. These are interesting for finding a low-level answer to the feasibility part of the path-finding problem. They can also be used as a navigation basis, but one must consider that depending on the robot dynamics, it may implicate an undesired behavior while navigating since these approaches loosely stipulate the path through the environment. Next, in Section 2.3, algorithms and techniques that consider the position and its derivatives for motion planning are presented.

2.3 Trajectory planning

2.3.1 Definition and characteristics

Trajectory planning is a problem defined as providing reference inputs to a robot control system to enable it to adopt a desired motion (GASPARETTO et al., 2012). In other words, trajectory planning determines *how* (position, velocity, and acceleration) to move along a path given the dynamics of the robot (LAVALLE, 2006).

The applications of UUVs and UAVs are not sufficiently well approximated by their kinematics. As a result, a planner who considers their dynamics is critical for developing a feasible and sound¹ trajectories. Additionally, the equations of motion allow for the computation of the energy and time consumption of trajectories. As Kong, Korukanti and Mettler (2009) demonstrates, performance criteria can affect the trajectories in a way that heuristic methods are not able to achieve. According to Gasparetto et al. (2015), three of the main optimality criteria for trajectory planning are: minimum execution time, minimum energy, and minimum jerk. The first one is applied when high productivity and reaction times are desired. The second is used whenever a smooth, low-stress, energy-saving trajectory is required. And the third one produces low-stress, resonance-reduced, smoother trajectories that are also easier to track (reduction of error).

Goerzen, Kong and Mettler (2010) states that 3-dimensional trajectory planning problems are significantly more difficult to solve due to the differential constraints. By considering the association between time and state space, finding an exact solution to even the most basic problems is not trivial and, generally, not possible. Additionally, adopting a heuristic or some method of approximation is usually necessary to both find a feasible trajectory and match the algorithm to the hardware capacity.

In the following sections, some of the algorithms that are used for UAVs' and UUVs' trajectory planning are presented.

2.3.2 Trajectory planning algorithms

According to Goerzen, Kong and Mettler (2010), trajectory planning algorithms can be divided into five major categories: sampling-based, decoupled trajectory planning, finite-state motion model, mathematical programming, and receding horizon control. In this subsection, each of these categories is presented together with some of the key characteristics.

2.3.2.1 Sampling-based trajectory planning

Donald et al. (1993) proposed one of the first trajectory planning algorithms that establishes proof of the resolution completeness and optimality of a polynomial approxi-

¹A sound trajectory is the one that guarantees the vehicle will enter the goal region and stay there without hitting any obstacles, despite uncertainties in sensing and control (GOERZEN; KONG; METTLER, 2010).

mation to the problem with kinodynamic constraints. This approach, called Grid-Based State Space Search, also has one of the few explicit safety guarantees: an arbitrary speed-varying safety corridor. However, this method also requires discretizing the entire state space into a lattice that is searched for the time-optimal path, making it impractical for real-time implementations. Some of the techniques derived from this method apply nonuniform discretizations to reduce the algorithm's complexity.

As described by Bertsekas (2012), the State-Space Navigation Function with Interpolation method also approximates the time-optimal path to a goal for a discrete state space. However, it returns a navigation function over the state space that can be computed either by value iteration or by control policy iteration. By following a gradient descent on this function, an approximately time-optimal trajectory is produced. Additionally, the interpolation between the points of the discrete space allows for a feedback continuous function.

Another method, introduced by LaValle et al. (1998), is the Rapidly-Exploring Random Trees (RRT) algorithm. By using a stochastic search in a body-centered frame of reference, this approach explores the configuration space. The classic implementation of this algorithm produces a solution to the kinodynamic planning problem that is probabilistic complete. However, it has no proof of optimality or convergence rate. This stochastic approach may face difficulties with long passageways, for example, needing to be associated with locally optimal techniques to achieve good performances. There is a vast literature on RRTs, making it one of the most significant references to kinodynamic planning.

A different method that uses a body-centered frame of reference is the Reachability Graph technique. It explores the state space using a tree that reaches even the maximum deflection states. This method, applied by LaValle (2006) to a 2-dimensional, curvature-constrained path problem, has exponential complexity that makes it impractical. In order to achieve better performance, one can use cell-based pruning so that a tree has no more than one leaf ending in each cell.

2.3.2.2 *Decoupled trajectory planning*

As mentioned before, some of these algorithms can use (as input) a discrete path in the $\mathcal{X}$ space to achieve a feasible dynamic-constrained trajectory (GOERZEN; KONG; METTLER, 2010). It is the case of the Minimum Distance Discrete Path Followed by the Trajectory Forming method. This allows for the efficient computation of approximate solutions but increases the difficulty of proving the completeness, optimality, and soundness of these algorithms. Additionally, soundness requires re-computing/rejecting unfeasible trajectories, and to avoid these, the implementations often use conservative safety corridors.

In the Discrete $\mathcal{X}$ space Search Connected by Two-Point Free-Space Boundary Value

Solver approach, a set of waypoints is selected, then a velocity is assigned to each one, and a boundary-value problem connecting these points is solved. Generally, the solution of this method is approximated using numerical methods, and an explicit collision check is necessary to ensure the soundness of the solution (GOERZEN; KONG; METTLER, 2010).

A strategy that is applied in most practical implementations is called Hierarchical Decoupled Planning and Control. This approach involves open-loop and closed-loop controllers linked together. These controllers: search for waypoints to the goal while avoiding obstacles; smooth the path so that it becomes feasible given the vehicle's kinodynamic limits; generate a set point timing function that moves through space; and track the set point while trying to minimize the associated error. This provides a straightforward way of integrating a waypoint planning algorithm with the vehicle control system since many unmanned aircraft are often already equipped with an inner-loop tracking controller (GOERZEN; KONG; METTLER, 2010).

The Discrete $\mathcal{X}$ space Search Interpolated with Polynomial Arcs tries to produce optimized trajectories by fitting polynomial arcs into waypoints produced by a discrete planner (GOERZEN; KONG; METTLER, 2010). Additionally, when coupled with the previous method, the Curvature-Constrained Path with Speed-Control Planning approach can produce time-optimal trajectories. This is achieved by taking a curvature-constrained path and optimizing the speed along the path.

2.3.2.3 *Finite-state motion model*

Since one of the main planning challenges for mobile robots is high-dimensional state space, this class of algorithms focuses on reducing search or optimization problems into finite ones. Thus, they are able to significantly reduce the solution's computational complexity. These algorithms are mainly in two classes: discrete-time models with quantized states and discrete-stage operations with fixed start and end states (GOERZEN; KONG; METTLER, 2010).

2.3.2.4 *Mathematical programming*

Mathematical programming methods are known as numerical optimization approaches. They are capable of finding resolution-optimal trajectories. However, their cost functions typically have local minima, and therefore, they rely significantly on initial guesses. Enforcing the equations of motion as constraints, and discretizing the underlying variational principles of the systems' dynamics to be used as constraints, are some of the strategies used by these methods.

The Infinite Horizon Control method creates an initial trajectory using some simplification (e.g., constant velocity) and performs a discrete search over the $\mathcal{X}$ space. Then, this trajectory is used as the first step for the mathematical programming search (TOUS-

SAINT; BASAR; BULLO, 2001).

2.3.2.5 *Receding horizon control*

The Receding Horizon Control approach (JADBABAIE; YU; HAUSER, 1999), as the name implies, solves the numerical optimization for a reduced time frame. Therefore, it requires less computational time. However, its optimality and completeness are hard to prove, and the globally optimal solution demands an appropriate cost function that enables capturing the rejected part of the trajectory. This technique is compatible with many UAV applications (GOERZEN; KONG; METTLER, 2010), since it can deal with sensory uncertainty and only local information is integrated. One of the limitations of this method is that it cannot be applied to highly agile vehicles.

2.4 **Learning-based navigation**

It is important to mention that in the last decades, there has been a significant improvement in learning-based navigation techniques. Since simulation and data storage and processing have evolved significantly, these techniques are achieving remarkable results when compared to classic model-based algorithms. However, most of the results are based on simulations or controlled environments. In the next few paragraphs, some state-of-the-art works are presented as a representation of these modern learning-based techniques.

In the work by Palossi, Conti and Benini (2019), the authors present a closed-loop visual pipeline for autonomous navigation based on a deep-learning algorithm. In the experiments, a nano-size UAV running a fully-onboard system was employed. The results demonstrate the high responsiveness of the system by avoiding collisions with dynamic obstacles and the fully autonomous indoor navigation of the system by traversing a 113m unknown environment.

Zhang et al. (2020) presents a deep framework that approaches AUV navigation as a deep sequential learning problem. This method takes raw sensor data at different frequencies as input, then utilizes a simplified attention mechanism and Fully Connected layers to output velocities, constructing the AUV navigation by accumulating these. The authors do not use model building or state estimation. Also, they provide comparisons between the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) methods and their approach by running an AUV platform in the sea.

In the work by Guo et al. (2020), an autonomous path planning model based on deep reinforcement learning is proposed for ship navigation in an unknown environment. This model uses a deep deterministic policy gradient (DDPG) combined with the artificial potential field method. Also, it uses continuous interaction with the environment and historical experience data, thus learning the optimal action strategy in simulation. Finally,

the authors validated their model through comparative experiments.

As one may notice, the use of learning-based methods for autonomous robot navigation is present in both aerial and aquatic vehicle work, with promising results in both media. However, these approaches are data-dependent, i.e., they rely on data acquisition from simulations or physical experiments. This may be another challenge for the new branch of Robotics that is HUAUVs, since there are few prototypes that can perform the seamless transition and navigation in both media, and the simulated models still adopt simplifications in order to compute the vehicle physics.

2.5 Hybrid planning and navigation

As previously mentioned, multirotor-based HUAUVs lack studies on Trajectory Planning. However, there are recent works that address similar problems to the one discussed in this work.

Su et al. (2021) addresses the trajectory optimization problem for an HUAUV navigating between air and water and performing a hovering task. The authors describe the motion characteristics of the vehicle aerial-underwater navigation, derive its dynamic equations, and establish a mathematical model of trajectory optimization considering constraints on control variables and states in different media. The goal is to minimize the weighting sum of terminal error of position and velocity of the UAAR. An improved teaching- and learning-based optimization (ITLBO) algorithm is proposed to solve the optimization problem, and detailed procedures are elaborated. The study's simulated results demonstrate the effectiveness of ITLBO algorithm in solving trajectory optimization problems for HUAUVs in

In the work by Liang, Liu and Zeng (2021), the authors propose a novel heuristic generalized extensive neighborhood search GLNS-k-means algorithm for a multirotor, underwater-glider, and tail-sitter HUAUV (LU et al., 2021). The k-means technique is applied in information clustering, and the heuristic GLNS algorithm is applied in finding the best path according to path budgets and motion constraints. The authors also introduce an efficient online replanning strategy to adapt the trajectory according to dynamic obstacles. Finally, numerical MATLAB simulations using real-world terrains and current data are used for evaluating the proposed approach.

Wu et al. (2020) propose an improved teach and learn-based optimization (ITLBO) algorithm to obtain the trajectory with minimum terminal position error for a coaxial eight-rotor HUAUV. The authors used a model of navigation information and terrain matching in the trajectory optimization model to reflect the influence of underwater navigation error on the quality of the trajectory. The collision detection and cost functions were fitted to the inaccurate navigation information. The authors concluded through simulation that this trajectory optimization approach can reflect the real situation of a multi-phase task since it

is capable of calculating the collision probability between the vehicle and obstacles when GPS is unavailable underwater.

As presented, there are many approaches for planning the trajectory of mobile robots. However, three-dimensional multi-domain trajectory planning, especially for multirotor HUAUVs, is not yet covered in the literature. Some challenges, such as the usually high-dimensional state space and the abrupt environment transition, are significant to many techniques. Planning how to navigate through the state space can require assumptions and/or custom approaches and certain knowledge about the robot dynamic model in order to find the best trajectories. Therefore, conceiving a trajectory planner for a HUAUV is a significant and relatively new challenge, which can be dealt with by the use of appropriate tools. Next, in Section 3, the trajectory planning techniques and the robot model considered in this work are presented with the tools to verify its performance.

2.6 Summary of literature review and implications for practice

Trajectory planning is a field that applies many techniques to find a feasible trajectory from a starting point to an ending one. These techniques, as presented, can be grouped by some of their key characteristics. However, one can find many studies that combine one or more of these groups.

As discussed in this work, planning a trajectory for hybrid unmanned aerial-underwater vehicles is not trivial. Finding a path in a hybrid, static, and known environment can be done quite simply. Nevertheless, the operation dynamics of this vehicle class can pose significant constraints in tracking the planned paths.

Therefore, some strategies such as finite-state motion modeling and mathematical programming may require complex formulations and more computation time. Other strategies like receding horizon control can be computationally efficient, however, planning optimality may be hard to be achieved.

Two of the five presented groups are significantly powerful with respect to high-dimensional domains and complex dynamics. Sampling-based algorithms can efficiently explore the possibilities in this kind of problem, and decoupled trajectory planning can construct trajectories based on faster-calculated paths.

As mentioned before, combining both of these approaches may yield a powerful solution for HUAUV trajectory planning. For example, a path can be found using a sampling-based algorithm that leads from an underwater position to an aerial one while avoiding an obstacle. Applying a decoupled trajectory planning strategy, a way to traverse this path can be calculated considering its dynamics and approximating the real behavior of the vehicle.

Finally, even though some works in the literature utilize learning techniques, this work considers that many of them are concerned with the final product of the planning step: its

execution. However, well-developed planning strategies such as the one mentioned above could also benefit from the learning process in order to improve its efficiency while still providing a more compartmentalized approach. Therefore, this work focuses on formulating and experimenting with classical trajectory planning approaches and analyzing their results and capabilities.

3 METHODOLOGY

As stated in Problem 1, the goal of this work is to plan the motion of a HUAUV through a known, hybrid, partially blocked environment, aiming at minimizing the energy consumption of the vehicle. In the next sections, the chosen methodology for dealing with this problem is presented. In section 3.1, the equations for the vehicle model and control are presented concerning each medium and the transition process; then, in section 3.3, the trajectory planning algorithms are presented; and finally, in section 3.4, the metrics used for evaluating the planning algorithm performance are presented.

3.1 Hydrone vehicle model

The Hydrone vehicle is a HUAUV that has four vertical aerial actuators, two horizontal, forward-facing underwater actuators, and two vertical underwater actuators (Figure 12). Therefore, during flight, the vehicle model is the same as a UAV model. However, underwater, the Hydrone vehicle has an underactuated, ROV-like model. Hence, it relies on the passive stability of the roll motion, achieved by carefully placing the vehicle's center of buoyancy above the center of mass. Additionally, it can produce forward, upward, and downward forces, and pitch and yaw rotations (HORN et al., 2020).

3.1.1 Aerial dynamics and control

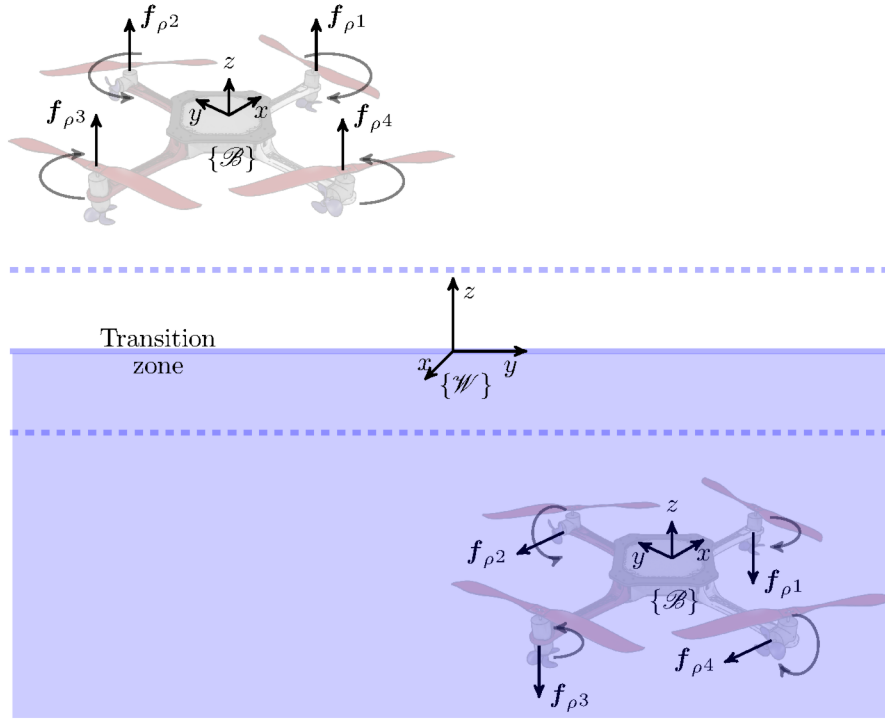
In this subsection and the two following ones the aerial, underwater, and transition models are presented.

First and foremost, being $\mathbf{x} \in \mathbb{R}^{12}$ the state vector and $\mathbf{u} \in \mathbb{R}^4$ the input vector, we have:

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} & \boldsymbol{\eta} & \mathbf{v} & \boldsymbol{\omega} \end{bmatrix}^T \text{ and } \mathbf{u} = \begin{bmatrix} \delta_T & \delta_\phi & \delta_\theta & \delta_\psi \end{bmatrix}^T,$$

where $\mathbf{p}$ and $\mathbf{v}$ are the vehicle's center of mass position vector and the velocity vector, respectively. Both values are relative to the global reference frame $\{\mathcal{W}\} \in \mathbb{R}^3$. Also, $\boldsymbol{\omega}$ represents the angular velocity vector in the body's reference frame $\{\mathcal{B}\}$, which is positioned at the center of mass of the HUAUV. Finally, $\boldsymbol{\eta}$ is the orientation vector in the Lie Group $\text{SO}(3)$, relative to $\{\mathcal{W}\}$. Simultaneously, the inputs δ_T , δ_ϕ , δ_θ , and δ_ψ are

Figure 12: Hydrone forces and moments



Above the vehicle operates in the aerial mode, while below it acts in aquatic mode. Around the surface level ($z = 0$) there is a transition zone.

functions describing the influence of the rotor speed vector $\Omega \in \mathbb{R}^4$ on the vertical thrust force and angular roll, pitch and yaw moments, respectively.

The final vehicle's dynamics can be written as follows:

$$\dot{\mathbf{p}} = \mathbf{v}, \quad (1)$$

$$m_\rho \dot{\mathbf{v}} = -\mathbf{R}_\eta \mathbf{C} |\mathbf{v}| \mathbf{v} - \boldsymbol{\omega} \times m \mathbf{v} - m \mathbf{g} + \mathbf{R}_\eta \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^4 \mathbf{f}_{\rho i} \end{bmatrix}, \quad (2)$$

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} \cos \theta & 0 & -\cos \phi \sin \theta \\ 0 & 1 & \sin \phi \\ \sin \theta & 0 & \cos \phi \cos \theta \end{bmatrix} \boldsymbol{\omega}, \quad (3)$$

$$\mathbf{J}_\rho \dot{\boldsymbol{\omega}} = -\mathbf{N} |\boldsymbol{\omega}| \boldsymbol{\omega} - \boldsymbol{\omega} \times \mathbf{J} \boldsymbol{\omega} + l \begin{bmatrix} \mathbf{f}_{\rho 2} - \mathbf{f}_{\rho 4} \\ \mathbf{f}_{\rho 3} - \mathbf{f}_{\rho 1} \\ -\sum_{i=1}^4 (-1)^i \mathbf{f}_{\rho i} \end{bmatrix}, \quad (4)$$

where $m_\rho = m + m_a(\rho)$ represents the sum of the vehicle's mass and the added mass effect, respectively. Additionally, $\mathbf{g}$ is the gravity vector, and $\mathbf{R}_\eta \in \text{SO}(3)$ denotes Euler's rotation matrix. The sum of the vehicle's tensor of inertia and the added inertia effect are

given by $\mathbf{J}_\rho = \mathbf{J} + \mathbf{J}_a(\rho)$. Since the added mass is a phenomenon that happens when the vehicle is underwater, while in the air: $m_\rho \approx m$ and $\mathbf{J}_\rho \approx \mathbf{J}$.

Matrices $\mathbf{C}$ and $\mathbf{N}$ represent the drag coefficients for translational and rotational motion represented by

$$\mathbf{C} = \frac{\rho}{2} \begin{bmatrix} \nabla^{2/3} C_u & 0 & 0 \\ 0 & \nabla^{2/3} C_v & 0 \\ 0 & 0 & \nabla^{2/3} C_w \end{bmatrix}, \text{ and}$$

$$\mathbf{N} = \frac{\rho}{2} \begin{bmatrix} \nabla^{5/3} C_p & 0 & 0 \\ 0 & \nabla^{5/3} C_q & 0 \\ 0 & 0 & \nabla^{5/3} C_r \end{bmatrix},$$

respectively (CONTE; SERRANI, 1996).

Finally, $\mathbf{f}_{\rho i} = \rho \zeta_\rho \boldsymbol{\Omega}_i^2$, $\forall i = 1 \dots 4$, are the forces generated by the vehicle's propellers running with $\boldsymbol{\Omega}_i$ speed, and rotation moments are dependent of the distance between the rotor and the vehicle's center of mass l . These forces are proportional to the *thrust coefficient* ζ_ρ , which is a function of the geometric parameters of the propeller and physical characteristics of the medium. It is important to observe that both, forces and moments, are dependent on the environment-specific mass ρ .

To stabilize the vehicle in the aerial operation mode, it is used nonlinear or linearized control laws at the operating point $\{\phi, \theta\} \approx 0$ (DREWS; NETO; CAMPOS, 2014). In such a case, we can define the speed of aerial propellers as:

$$\boldsymbol{\Omega} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix} \mathbf{u}. \quad (5)$$

As previously said, $\mathbf{u}$ represents the unbalancing of forces and moments of equations (2) and (4), respectively, therefore,

$$\delta_T = \frac{\lambda_p e_z + \lambda_d \dot{e}_z + \lambda_i \int e_z}{\cos \phi \cos \theta} + \sqrt{\frac{m \|\mathbf{g}\|}{4\rho \zeta_\rho}}, \quad (6)$$

is the nonlinear control law used for stabilizing the robot along the z -axis, where the error $e_z = z_{\text{ref}} - z$, λ_p , λ_i and λ_d are PID control gains, and $\rho \approx 1.23 \text{ kg/m}^3$ is the air specific mass. Also, we can define:

$$\begin{bmatrix} \delta_\phi \\ \delta_\theta \\ \delta_\psi \end{bmatrix} = \lambda_p \begin{bmatrix} e_\phi \\ e_\theta \\ e_\psi \end{bmatrix} + \lambda_d \begin{bmatrix} \dot{e}_\phi \\ \dot{e}_\theta \\ \dot{e}_\psi \end{bmatrix} + \lambda_i \int \begin{bmatrix} e_\phi \\ e_\theta \\ e_\psi \end{bmatrix}, \quad (7)$$

as PID controllers for the robot's attitude, where the error $e_\phi = \phi_{\text{ref}} - \phi$, $e_\theta = \theta_{\text{ref}} - \theta$ and $e_\psi = \psi_{\text{ref}} - \psi$. To move the robot along y and x -axes, it is used ϕ_{ref} and θ_{ref} , respectively, such that:

$$\begin{bmatrix} \phi_{\text{ref}} \\ \theta_{\text{ref}} \end{bmatrix} = \lambda_p \mathbf{R}_\eta \begin{bmatrix} e_x \\ e_y \end{bmatrix} + \lambda_d \mathbf{R}_\eta \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix}, \quad (8)$$

with the errors $e_x = x_{\text{ref}} - x$ and $e_y = y_{\text{ref}} - y$. In this context, one can operate the UAV in closed-loop by setting the reference vector:

$$\mathbf{r} = \begin{bmatrix} x_{\text{ref}} & y_{\text{ref}} & z_{\text{ref}} & \psi_{\text{ref}} \end{bmatrix}^T.$$

3.1.2 Underwater dynamics and control

Next, we described the mathematical model for the aquatic operation mode. Since the vehicle operates as a typical ROV (GOMES et al., 2005), we must define a new input vector $\mathbf{u} \in \mathbb{R}^4$, such that:

$$\mathbf{u} = \begin{bmatrix} \delta_T & \delta_V & \delta_\theta & \delta_\psi \end{bmatrix}^T,$$

where now δ_V represents the forward thrust.

In the water, the complete dynamics of the robot can be written as:

$$\dot{\mathbf{p}} = \mathbf{v}, \quad (9)$$

$$m_\rho \dot{\mathbf{v}} = -\rho \mathbf{R}_\eta \mathbf{C} |\mathbf{v}| \mathbf{v} - \boldsymbol{\omega} \times m \mathbf{v} - \mathbf{g}_\rho + \mathbf{R}_\eta \begin{bmatrix} \mathbf{f}_{\rho 2} + \mathbf{f}_{\rho 4} \\ 0 \\ \mathbf{f}_{\rho 1} + \mathbf{f}_{\rho 3} \end{bmatrix}, \quad (10)$$

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} \cos \theta & 0 & -\cos \phi \sin \theta \\ 0 & 1 & \sin \phi \\ \sin \theta & 0 & \cos \phi \cos \theta \end{bmatrix} \boldsymbol{\omega}, \quad (11)$$

$$\mathbf{J}_\rho \dot{\boldsymbol{\omega}} = -\rho \mathbf{N} |\boldsymbol{\omega}| \boldsymbol{\omega} - \boldsymbol{\omega} \times \mathbf{J} \boldsymbol{\omega} - \mathbf{c}_\rho + l \begin{bmatrix} 0 \\ \mathbf{f}_{\rho 3} - \mathbf{f}_{\rho 1} \\ \mathbf{f}_{\rho 2} - \mathbf{f}_{\rho 4} \end{bmatrix}, \quad (12)$$

where the water density is $\rho \approx 10^3$. Also, $\mathbf{g}_\rho$ and $\mathbf{c}_\rho$ are terms that incorporate gravity and buoyancy forces to provide underwater stability, in such a way that the resulting gravity force is:

$$\mathbf{g}_\rho = (m - \rho V) \mathbf{g}, \quad \text{such that } \rho V > m,$$

where buoyancy is slightly greater than weight, and

$$\mathbf{c}_\rho = c(m + \rho V) \|\mathbf{g}\| \begin{bmatrix} \sin \phi \\ \sin \theta \\ 0 \end{bmatrix},$$

is the passive stability moment. These two are both dependent upon the density ρ of the medium in which the robot with volume V navigates. A restoring moment is ensured by correctly positioning the centre of mass (where the gravity vector $\mathbf{g}$ acts) at a distance c of the centre of buoyancy.

In multi-rotor systems, the unbalanced forces and moments generated by the vehicle's thrust systems are the sources of movement. This characteristic is the same in different environments. However, the control strategies of aquatic robots such as ROVs are better feasible in the water environment. Both are discussed as follows.

In the underwater operation mode, the force provided by the set of propellers is divided into horizontal and vertical components. Therefore, the speed in the motors is now ruled by:

$$\mathbf{\Omega} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \mathbf{u}. \quad (13)$$

Concerning actuation forces and moments, we use the following control law to stabilize the depth of the robot:

$$\delta_T = \frac{\lambda_p e_z + \lambda_d \dot{e}_z + \lambda_i \int e_z}{\cos \phi \cos \theta} - \sqrt{\frac{\|\mathbf{g}_\rho\|}{2\rho\zeta_\rho}}. \quad (14)$$

Meanwhile, attitude moments can be stabilized by control laws similar to those in Eq. (7),

$$\begin{bmatrix} \delta_\theta \\ \delta_\psi \end{bmatrix} = \lambda_p \begin{bmatrix} e_\theta \\ e_\psi \end{bmatrix} + \lambda_d \begin{bmatrix} \dot{e}_\theta \\ \dot{e}_\psi \end{bmatrix} + \lambda_i \int \begin{bmatrix} e_\theta \\ e_\psi \end{bmatrix} dt, \quad (15)$$

where δ_ϕ is assumed to be null since roll moment is self-stabilized. In this configuration, the robot behaves similarly to a nonholonomic airplane, in a way that its position can be stabilized by a simple kinematic nonlinear control law:

$$\delta_V = \lambda_v d + \lambda_\alpha e_\psi^2, \quad (16)$$

where d and ψ_{ref} are the Euclidean distance and the orientation angle between $[x_{\text{ref}}, y_{\text{ref}}]$ and $[x, y]$, respectively. Similarly to the aerial case, we can define:

$$\mathbf{r} = \begin{bmatrix} x_{\text{ref}} & y_{\text{ref}} & z_{\text{ref}} & \psi_{\text{ref}} \end{bmatrix}^T.$$

3.1.3 Transition model

In the work by Ma, Feng and Yang (2018), the authors propose a linear variation for buoyancy, added mass, and drag force terms during the transition region. Additionally,

they consider the linearization errors as disturbances to be addressed by the vehicle control system. In (RAVELL; MAIA; DIEZ, 2018) the transition is considered by including the density as an additional system state.

Notably, specific masses and immersed volume significantly influence environment transitions. Of the two, adjusting the underwater volume appears most feasible. However, evaluating the volume accurately requires a detailed vehicle geometry description and simulation, as previously addressed by Aoki et al. (2022a). Unfortunately, this aspect has not been addressed in the current work.

Instead, a different method is suggested: adjusting the environment's specific mass to indirectly alter volume-dependent parameters. To accomplish this, a first-degree function is proposed for the specific mass within a specific range of height and depth, known as the Transition Zone.

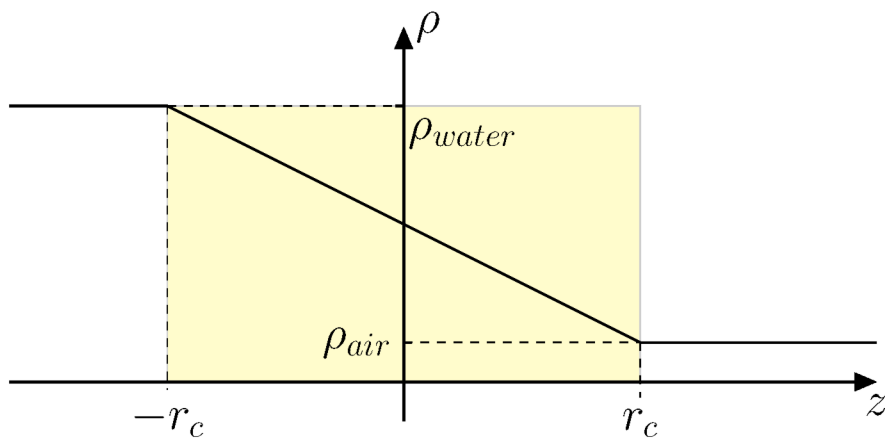
This model uses the vehicle's center hull radius r_c as the boundary limits of the Transition Zone. This approach is adopted because the most significant vehicle volume portions that are immersed during the transition are the main hull and the arms, where the first one initiates contact with the water.

The proposed specific mass function considering the environment transition is shown in Fig. 13 and can be represented by the following equation:

$$\rho(z) = \begin{cases} \rho_{water}, & \text{if } z < -r_c \\ -\frac{\rho_{water}-\rho_{air}}{2r_c}(z - r_c) + \rho_{air}, & \text{if } -r_c \leq z \leq r_c \\ \rho_{air}, & \text{if } z > r_c \end{cases} \quad (17)$$

where ρ_{air} and ρ_{water} are the air and water-specific masses, respectively.

Figure 13: The specific mass function for the environment transition.



3.1.4 Vehicle constraints

As presented, the HUAUV in question has some limitations. The main one is in relation to its underwater performance. This model, proposed by Horn et al. (2020), has a pair of propellers facing forward as a way of acting in x -axis on the body frame. Still, it does not have actuators facing backward. Thus, as this vehicle model was developed considering off-the-shelf products, its motor-propeller sets do not consider inversion of rotation. Therefore, the vehicle depends on the drag force to reduce its speed underwater.

As the vehicle's motor-propeller combinations do not have a reverse. Its underwater maneuverability is significantly impacted. First, the vehicle cannot rotate around the z -axis in the body frame. Second, when it describes curves underwater, the radius of curvature of these curves depends on their momentum. In Section 3.3 the chosen approach for dealing with this kind of complex nonlinear dynamics is presented.

3.2 Energy consumption

Mobile robots are usually powered by batteries. Although there are solutions such as umbilical cables and the use of solar panels, these alone also have their limitations, e.g., the need for large, open spaces, and working outdoors with good weather conditions. As this work focuses on energy-efficient planning, this session presents what this work refers to with the word energy and the impact and limitation of the model followed.

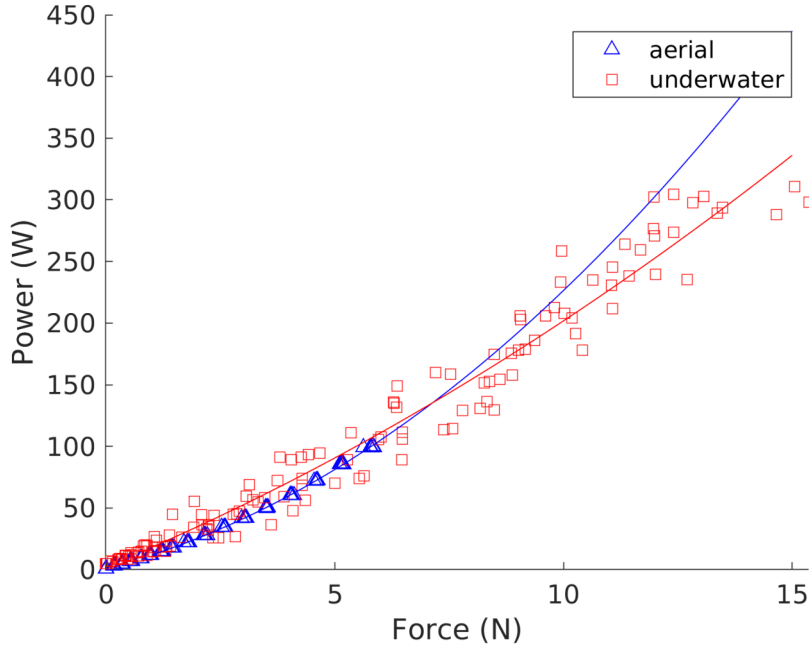
As proposed in the work by Bauersfeld et al (BAUERSFELD; SCARAMUZZA, 2022). it is not only possible to work with a battery model, but it is also possible to predict optimal travel speeds that increase the area covered by the vehicle. However, this refers to a situation where there is no speed and safety limitation. To operate a vehicle in a safe environment with obstacles, it is necessary to limit the speed of the vehicle and to plan its trajectory efficiently.

3.2.1 Power estimation

In the work by Horn et al. (HORN et al., 2019) the authors carried out a survey of energy consumption for some propeller-and-motor combinations for HUAUVs. By having access to this data, it was possible to extract polynomial curves of energy consumption that associate it with the forces generated by the sets of motors and propellers. Such curves could be used in this work since the combinations adopted in the already presented vehicle model are among those raised by the authors.

The curves used in this study are not exact representations of the vehicle's energy consumption, as they only consider the motor consumption. Nevertheless, this work assumes that the motor's energy use is significant enough to represent the overall energy consumption of a vehicle in a hybrid environment. Moreover, since the motors have higher power than other system components (e.g., sensors and microcontrollers), their consumption is

Figure 14: This figure shows the power consumption and polynomial regression curves for aerial and underwater actuation systems. Both curves follow a second-degree polynomial trend. The underwater system presents higher consumption than the aerial system from null force until $7.5N$, while the aerial system assumes higher values for higher forces, widening the difference between the two systems.



considered representative of the overall energy usage.

3.3 Trajectory planning algorithms

The effectiveness of motion planning algorithms is determined by various factors, such as the vehicle model, the level of environmental dynamics, and the extent of prior knowledge of the navigation domain. Due to the application-specific nature of many of these algorithms, customized approaches are often necessary. Nonetheless, some more versatile algorithms, such as those based on Rapidly-Exploring Random Trees (RRTs), can accommodate dynamic and partially known environments while allowing for continuous planning.

In this regard, the current work proposes an RRT-based trajectory planning approach named Hybrid Closed-loop RRT, which takes into account the constraints of a hybrid environment and provides a suitable trajectory for the vehicle to follow. Additionally, two heuristics are studied to improve the algorithm's performance. These heuristics involve the use of constant estimated costs for each medium and for the transition, as well as the use of a skewed tree expansion.

3.3.1 Closed-loop RRT

One of the algorithms that use RRTs for trajectory planning is the one proposed by Kuwata et al. (2009), called Closed-loop RRT (CL-RRT), which implements real-time motion planning based on these trees by adding closed-loop prediction. The CL-RRT is an online planner, i.e., it plans as the vehicle progresses through the $\mathcal{X}_{free}$ space. This algorithm enables navigation in both fully and partially known environments. Additionally, the CL-RRT algorithm verifies the collision using a vehicle model to propagate a trajectory that is associated with the reference tree. Therefore, it can foresee collisions.

Like the other algorithms, the CL-RRT is initialized with a node at $\mathbf{x}(0)$, then it updates the vehicle's current state and the surrounding environment through its sensors. Afterward, it expands the tree and propagates the associated vehicle trajectory through the controllers until the time limit T is reached. Then, the current best trajectory in the planner is checked for collisions, and if it is clear, the trajectory is sent to the controller. However, if a trajectory is not calculated within the time limit, a safety action must be taken, and if a collision is found in the trajectory, the unfeasible parts must be trimmed out of the reference tree and a new trajectory must be chosen.

3.3.2 Hybrid Closed-loop RRT

The CL-RRT algorithm (KUWATA et al., 2009) can be used in several situations. Among these, for trajectory planning of unmanned aerial vehicles (ARSLAN; BERN-TORP; TSIOTRAS, 2017; ZHU; CHENG; LI, 2018). To this end, the algorithm uses the vehicle model to generate trajectories in the $\mathcal{X}$ space and verify possible collisions. Then, these trajectories are associated with nodes of the exploration tree.

However, to be used for optimal planning, as in the RRT* algorithm (KARAMAN; FRAZZOLI, 2010), it is necessary to intensively use the model to generate multiple trajectories in space and then select the best among them. With this, a relevant question arises about the algorithm execution time for the non-linear vehicle model. As these models can have significant execution times, we adopted a methodology for optimization that does not generate trajectories for all nodes of the expanded tree, but only for those selected as best and that can be sent to the low-level controller.

In order to develop a planner that did not need to solve the vehicle dynamics equations for each node of the expanded tree, a survey of the navigation costs associated with each environment and at transition was carried out.

In this survey, significant variance in the data was verified, however, it was possible to observe that the water navigation cost is much lower than the air navigation cost. Thus, two techniques were proposed for optimizing the energy efficiency of HUAUVs:

1. the use of constant estimated costs for each medium, and for the transition;
2. and the use of a skewed tree expansion.

Another significant aspect of Hybrid CL-RRT is that it should be able to handle domain transition. As the vehicle model and several others in the literature focus on vertical medium transition, this planner maintains this approach to avoid possible disturbances and instabilities during vehicle navigation. In this way, this proposed planner also seeks to approximate the real application of this class of vehicles.

The main loop of the hybrid planner is presented in Algorithm 1. Essentially, this is the same as that adopted in the work by (KUWATA et al., 2009). In this high-level view of the planner, it can be seen that it has an initial phase of updating both the vehicle state and the map. Then, the propagation of the vehicle trajectory is made for the next instants, predicting its state after the planning. Then, the hybrid tree is expanded, and the best path is selected. Finally, the repropagation procedure is performed, where the references associated with the best path are passed to the predictive model and the approximate trajectory to be performed by the vehicle is identified. If this trajectory is valid, then the references are passed to the navigation system, otherwise, the invalid nodes are marked, before starting a new loop.

Algorithm 1 Hybrid CL-RRT execution loop

Require: $x(0)$, r_{goal}

```

1: initialise  $\mathcal{T}$  with node at  $x(0)$ 
2: repeat
3:   update  $x(t)$  and  $\mathcal{X}_{\text{free}}(t)$ 
4:   propagate  $x(t)$  to  $x(t + T)$  with hybrid system model
5:   repeat
6:     expand  $\mathcal{T}$ 
7:   until time limit  $T$  is reached
8:   choose the current best path  $\mathcal{P} \in \mathcal{T}$ 
9:   if  $\mathcal{P} \in \emptyset$  then
10:    apply safety action and goto line 18
11:   end if
12:   re-propagate from  $x(t + T)$  by using references associated with  $\mathcal{P}$ 
13:   if  $x(t) \in \mathcal{X}_{\text{free}}(t) \forall t \in [t, t + T]$  then
14:     send  $\mathcal{P}$  to the controller
15:   else
16:     mark infeasible parts of  $\mathcal{T}$  and goto line 8
17:   end if
18:    $t \leftarrow t + T$ 
19: until reach  $r_{\text{goal}}$ 

```

The expansion of this hybrid tree is done similarly to that of a traditional tree: random points are sampled from the map; the tree is expanded in the direction of this point; if there is a collision in the edge between these two points, the new point is discarded, otherwise it is added to the tree; and so on. However, there is one important difference in this expansion: the medium transition.

Algorithm 2 Hybrid CL-RRT expansion

Require: Map, *segment_length*

```

1:  $\mathbf{p}_{sample} \leftarrow \text{sampleRandomPoint}()$ 
2:  $\text{closest\_node} \leftarrow \text{getClosestNode}(\mathbf{p}_{sample})$ 
3:  $\mathbf{p}_{closest} \leftarrow \text{closest\_node.pos}$ 
4:  $\mathbf{p}_{dir} \leftarrow \mathbf{p}_{sample} - \mathbf{p}_{closest}$ 
5:  $\mathbf{p}_{new} \leftarrow \mathbf{p}_{closest} + \hat{\mathbf{p}}_{dir} \cdot \text{segment\_length}$ 
6: if  $\text{transition}(\mathbf{p}_{closest}, \mathbf{p}_{new})$  then
7:    $\mathbf{p}_{new} \leftarrow \text{verticalTransition}(\mathbf{p}_{closest}, \mathbf{p}_{new})$ 
8: end if
9: if  $\text{collision}(\mathbf{p}_{closest}, \mathbf{p}_{new})$  then
10:   return
11: else
12:    $\mathcal{T}.addNode(\mathbf{p}_{new})$ 
13: end if

```

In order to implement a hybrid trajectory planning approach, it is essential to devise effective strategies for transitioning between different domains. In our study, we adopted a specific strategy that involves conditioning the transition between nodes on a vertical edge. Essentially, when a new point is sampled and the closest existing point is either in the transition region or in another medium, we adjust the x and y coordinates of the new point position to ensure that it satisfies the vertical condition. This way, we can facilitate a smoother and more efficient transition between different domains during the robot's trajectory planning.

Because this work is not yet concerned with strategies for the best medium transition, the strategy adopted only provides a form of transition. It is therefore assumed that this will be sufficient to satisfactorily generate paths that explore both domains. It is also assumed that the repropagation step (trajectory verification), before passing the trajectory to the navigation system, can predict collision situations well enough and that the HUAUV control can handle the references provided from these vertical transitions well enough.

3.3.2.1 Water-biased Hybrid CL-RRT

As verified in the energy consumption survey conducted by Horn et al. (2019), the energy consumption per unit of force is higher for the aerial than for the aquatic domain. Also, from the construction aspects of HUAUVs, it is known that these vehicles need to generate more force to keep the vehicle in the air than to keep it submerged. Thus, the first heuristic proposed to reduce energy consumption consisted in biasing the tree expansion to further explore the water environment. From this, it is expected that the vehicle will generate trajectories with lower amounts of force and consequently, less energy consumption for the same displacement.

As described earlier, the expansion of an RRT takes place through the random sam-

pling of points in space. Therefore, in order to bias this sampling, a technique was used in which pseudo-random numbers are generated in a certain interval. If the generated number is smaller than a constant percentage of the interval, then the point is sampled in the water domain, otherwise, it can be sampled anywhere in the hybrid space.

As shown in the line 1 of the Algorithms 2 and 3, the *sampleRandomPoint()* function is designed to operate impartially when no arguments are given. If one is given, then the function takes the percentage given as the proportion of points to be sampled from the water.

Algorithm 3 Water-biased Hybrid CL-RRT expansion

Require: Map, *segment_length*, *water_bias*

```

1:  $\mathbf{p}_{sample} \leftarrow \text{sampleRandomPoint}(\text{water\_bias})$ 
2:  $\text{closest\_node} \leftarrow \text{getClosestNode}(\mathbf{p}_{sample})$ 
3:  $\mathbf{p}_{closest} \leftarrow \text{closest\_node.pos}$ 
4:  $\mathbf{p}_{dir} \leftarrow \mathbf{p}_{sample} - \mathbf{p}_{closest}$ 
5:  $\mathbf{p}_{new} \leftarrow \mathbf{p}_{closest} + \hat{\mathbf{p}}_{dir} \times \text{segment\_length}$ 
6: if  $\text{transition}(\mathbf{p}_{closest}, \mathbf{p}_{new})$  then
7:    $\mathbf{p}_{new} \leftarrow \text{verticalTransition}(\mathbf{p}_{closest}, \mathbf{p}_{new})$ 
8: end if
9: if  $\text{collision}(\mathbf{p}_{closest}, \mathbf{p}_{new})$  then
10:   return
11: else
12:    $\mathcal{T}.addNode(\mathbf{p}_{new})$ 
13: end if
```

3.3.2.2 Estimated Costs Hybrid CL-RRT

The second proposed heuristic is based on the assumption that there are approximately constant average costs for traveling between two points in the same environment or the transition. Therefore, based on a survey of these average costs, it would be possible to estimate the cost associated with a path section (between two nodes). This way, it would be possible to identify, besides the nodes closest to a certain position, those which have the lowest cost in a certain region. Thus, besides simply adding child nodes to the closest nodes, it would be possible to add the new nodes to the least costly ones in a given region, in an attempt to minimize (at least locally) the cost of the paths.

The expansion function associated with this heuristic has a similar structure as the previous ones. However, as presented in Algorithm 4 at line 12, instead of simply adding the new node as a child of the closest node, it is made the verification of which parent node (neighbor) in a certain radius of the new node (neighborhood) would generate a lower cost displacement to it. After checking the best parent node, the new node is added to the tree as a child of that one. Then, since this node is now part of the tree, it is verified which nodes in the neighborhood can be reached with less cost. If a path is found that

allows reaching any of the neighbors at a lower cost, rewiring is performed.

It is clear that this approach would increase the number of steps for the expansion of the tree, which would probably lead to a longer execution time for each new insertion. However, this work seeks to evaluate the possibility of reducing energy consumption in missions performed by HUAUVs, more specifically the reduction of energy consumption by their motors, and not the reduction of execution time or consumption of computational resources.

Algorithm 4 Estimated Costs Hybrid CL-RRT expansion

Require: Map , segment_length , cost_{air} , cost_{uw} , $\text{cost}_{\text{trans}}$, r

```

1:  $\mathbf{p}_{\text{sample}} \leftarrow \text{sampleRandomPoint}()$ 
2:  $\text{closest\_node} \leftarrow \text{getClosestNode}(\mathbf{p}_{\text{sample}})$ 
3:  $\mathbf{p}_{\text{closest}} \leftarrow \text{closest\_node.pos}$ 
4:  $\mathbf{p}_{\text{dir}} \leftarrow \mathbf{p}_{\text{sample}} - \mathbf{p}_{\text{closest}}$ 
5:  $\mathbf{p}_{\text{new}} \leftarrow \mathbf{p}_{\text{closest}} + \hat{\mathbf{p}}_{\text{dir}} \times \text{segment\_length}$ 
6: if  $\text{transition}(\mathbf{p}_{\text{closest}}, \mathbf{p}_{\text{new}})$  then
7:    $\mathbf{p}_{\text{new}} \leftarrow \text{verticalTransition}(\mathbf{p}_{\text{closest}}, \mathbf{p}_{\text{new}})$ 
8: end if
9: if  $\text{collision}(\mathbf{p}_{\text{closest}}, \mathbf{p}_{\text{new}})$  then
10:   return
11: else
12:    $\text{min\_cost} \leftarrow \text{closest\_node.cost} + \text{getCost}(\mathbf{p}_{\text{closest}}, \mathbf{p}_{\text{new}})$ 
13:    $\text{min\_cost\_node} \leftarrow \text{closest\_node}$ 
14:    $\text{neighbours} \leftarrow \text{getNodesInRadius}(\mathbf{p}_{\text{new}}, r)$ 
15:   for  $\text{node} \in \text{neighbours}$  do
16:     if  $\text{transition}(\mathbf{p}_{\text{node}}, \mathbf{p}_{\text{new}})$  then
17:       continue
18:     else
19:        $\mathbf{p}_{\text{node}} \leftarrow \text{node.pos}$ 
20:        $\text{this\_cost} \leftarrow \text{closest\_node.cost} + \text{getCost}(\mathbf{p}_{\text{node}}, \mathbf{p}_{\text{new}})$ 
21:       if  $\text{this\_cost} < \text{min\_cost}$  then
22:          $\text{min\_cost} \leftarrow \text{this\_cost}$ 
23:          $\text{min\_cost\_node} \leftarrow \text{node}$ 
24:       end if
25:     end if
26:   end for
27:    $\mathcal{T}.\text{addNode}(\mathbf{p}_{\text{new}})$ 
28:    $\text{rewireTree}(\text{new\_node}, \text{neighbours})$ 
29: end if

```

To carry out the survey of cost estimates, several techniques could be used: from an experimental survey to consulting manufacturers' catalogs, or even using generic models for motors. However, for this work, it was considered that this survey could be carried out using a significant number (in the order of thousands) of small trajectories in simulation. It is clear that for this the motor model used in the simulation should be consistent and close to the real one, as is the case.

3.4 Performance of trajectory planning

Evaluating the performance of trajectory planning algorithms depends on what characteristics are desired from the planner. The present work is concerned with:

- Planning a path that tries to minimize energy consumption;
- Finding feasible hybrid trajectories for a HUAUV in an unknown environment.

Thus, some of the key metrics for evaluation are energy consumption, distance traveled, and travel time. For each test, it is verified if the vehicle was able to move from an initial point to the goal region, and then its performance is compared with and without the proposed heuristics, for the same scenario through these metrics.

Among the proposed metrics, the most relevant after verifying that the vehicle was able to reach its objective is the energy consumption metric. Thus, one can compare the effectiveness of the proposed planner. Furthermore, through distance traveled and travel time, the impact on vehicle navigation and possible trade-offs involved in its focus on energy efficiency can be assessed.

4 RESULTS

In this section, the results of the experiments carried out in the simulation are presented. First, in Section 4.1, the simulation tool adopted in this study is described. Second, in Section 4.2, the experimental setup is presented. Then, in Section 4.3 the results are shown and their main characteristics are emphasized. And finally, in Section 4.4 the results are analyzed and discussed.

4.1 Simulator

The algorithms and techniques of this study were evaluated using a previously-developed simulator (DREWS; NETO; CAMPOS, 2014; HORN et al., 2020) from Project Hydron of the Automation and Intelligent Robotics research group (NAUTEC). However, the simulator was not designed for path planning or trajectory planning algorithms, so it needed to be adjusted to meet the needs of this study.

This simulator was developed in a multi-paradigm programming language and for numeric computing environment software. Its need comes from the lack of an open-access simulator for hybrid aerial-underwater vehicles and it is mainly focused on vehicle dynamics and control.

The basic architecture of the customized simulation provides the planner with a map. This map is utilized by the planner to calculate a trajectory, which in its turn is provided to the navigation system that will pass the trajectory references to the low-level position controller (NETO et al., 2015).

4.1.1 World, map, and obstacles

This work's simulation uses simplified versions of world and map. Its world has only two kinds of spaces: spherical obstacles (occupied space), and empty space. Also, the air-water interface is a horizontal plane at $z = 0$ in the world frame. The map representation can be different from the world due to the unknown obstacles, which are included in the map once their surface is inside the vehicle perception distance. These simplified representations were chosen for the ease of algorithmic manipulation and not being the main

subject of this work. However, more realistic approaches should be sought to approximate the problem to real-world applications.

4.1.2 Parameters

In Table 2, the parameters associated with the simulation are presented. The first parameter, perception radius, refers to the maximum distance between the vehicle and the obstacle border so that this obstacle is added to the map. It was set to a value that approximates common sensor ranges for aerial and underwater perception. The world occupancy is the ratio between the total obstacle volume and the world volume. And the number of obstacles refers to the number of spheres in the world space. Both values were chosen to represent large, but avoidable obstacles. And the world dimensions show how the world space is distributed. These values were chosen to provide sufficient space for HUAUV maneuvers without overly increasing the $\mathcal{X}$ space.

Table 2: Simulation parameters

Parameter	Value
Perception radius	5 m
World occupancy	1%
Number of obstacles	10
World dimensions	20m x 20m x 20m

Figure 15: Example of a simulated world

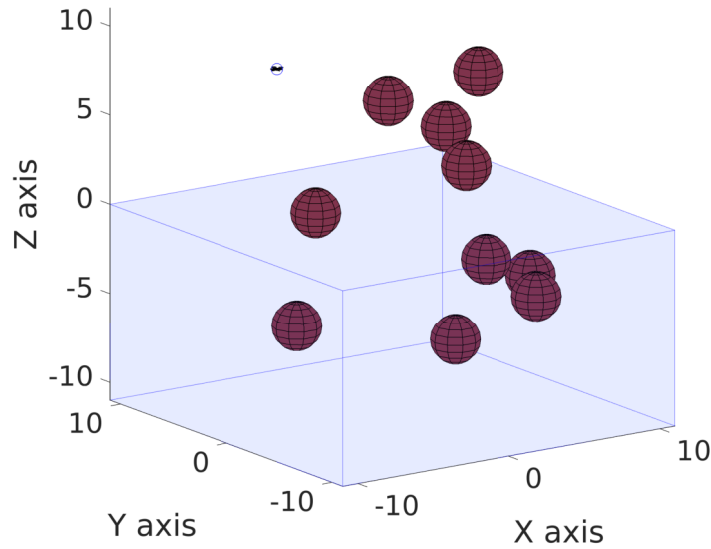


Figure 15 presents an example of a random scenario in the utilized simulator. As an example of a scenario, all the obstacles were set as discovered. Otherwise, only those inside the vehicle perception radius would be displayed.

4.2 Experimental setup

Simulation trials were executed using Matlab language on an Intel Core™ i5-3330 CPU 3 GHz x 4 and 16 GB of RAM under Ubuntu 20.04. In Table 3, 4, and 5, some of the parameters used in the simulation are presented. These were based on a vehicle concept called Hydrone, currently under development at the NAUTEC laboratory of FURG. Some of these parameters were obtained from previous work: the aquatic motor-propeller set is based on the study by Horn et al. (2019), and the values for the aerial motor-propeller set were extracted from the article by Maia, Mercado and Diez (2017).

Table 3: Parameters used in both media in the simulated experiments.

Parameter	Value
m [kg]	1.29
V [m ³]	1.60×10^{-3}
l [m]	0.27
r_c [m]	0.07
c [m]	0.02
g [m/s ²]	$\begin{bmatrix} 0 & 0 & 9.78 \end{bmatrix}$
J [kgm ²]	$\begin{bmatrix} 1.45 & 0.02 & -0.08 \\ 0.02 & 2.88 & -0.01 \\ -0.08 & -0.01 & 1.54 \end{bmatrix} \times 10^{-2}$
C	$\text{diag}[1.63, 1.63, 2.57] \times 10^{-2}$
N	$\text{diag}[0.47, 0.47, 0.95] \times 10^{-2}$

Table 4: Parameters used in the aerial environment ($\rho \approx 1.29 \text{ kg/m}^3$) in the simulated experiments.

Parameter	Value
m_ρ [kg]	1.29
J_ρ [kgm ²]	$\begin{bmatrix} 1.45 & 0.02 & -0.08 \\ 0.02 & 2.88 & -0.01 \\ -0.08 & -0.01 & 1.54 \end{bmatrix} \times 10^{-2}$
ζ_ρ	2.45×10^{-7}

4.2.1 Estimated energy consumption

To estimate the average energy consumption by the adopted model, a simulation scenario was developed in which it was explored by an RRT and, in each node of the tree, the

Table 5: Parameters used in the underwater environment ($\rho \approx 1000.00 \text{ kg/m}^3$) in the simulated experiments.

Parameter	Value
m_ρ [kg]	1.93
$\mathbf{J}_\rho$ [kgm ²]	$\begin{bmatrix} 1.56 & 0.14 & 0.04 \\ 0.14 & 3.00 & 0.11 \\ 0.04 & 0.11 & 1.66 \end{bmatrix} \times 10^{-2}$
ζ_ρ	1.74×10^{-9}

energy consumption of the parent node until the current node was stored. It also stored the distance traveled in the air, in the water, and the transition interface between each node. As the RRTs randomly explore the environment, it was only necessary to add the number of nodes to be inserted in the tree and propagate the model for each one. Finally, to mitigate bias for initial conditions of the tree, the different configurations of the same size scenario were performed six times, inserting, in each one of these, five thousand nodes, for a total of thirty thousand nodes with energy costs and distances collected.

The estimated energy consumption survey results are presented in Figures 16 and 17. According to the figures, it is clear that the approximately normal distributions of the performed survey present a higher average consumption for the aerial trajectories in comparison with the aquatic trajectories. Also, in the boxplots presented, it can be observed that the distributions for the trajectories through the air-water interface present similar consumption to the aquatic trajectories. The median values of the distributions were 337.91 J/m , 190.68 J/m , and 181.46 J/m for the average air, transition, and aquatic consumption, respectively.

4.3 Simulated results

As a way to test the heuristics and their ability to generate the least-cost trajectories, experiments were conducted in 200 different scenarios. In each scenario, the following were randomly altered: the arrangement of obstacles, the starting point, and the objective region. After that, a comparative analysis of the results of the planners that used heuristics and the original planner was performed.

As the purpose of these tests was to analyze the most diverse situations with each planner, the initial and final states were slightly influenced, only having null velocities. This allowed for obstacles to be significantly close to the vehicle since the beginning of its operation. This, as well as the fact that the vehicle had no way to brake underwater, led to some unsuccessful missions, resulting in the vehicle not being able to reach its goal safely. Thus, the results for the number of successful missions for each planner were as

Figure 16: The energy consumption distribution histograms for each medium. In the vertical axis are the number of samples for each interval.

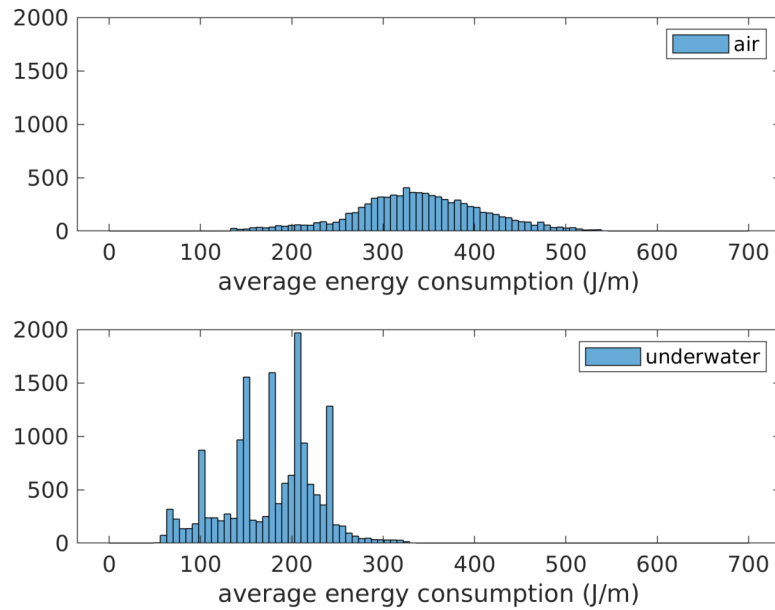
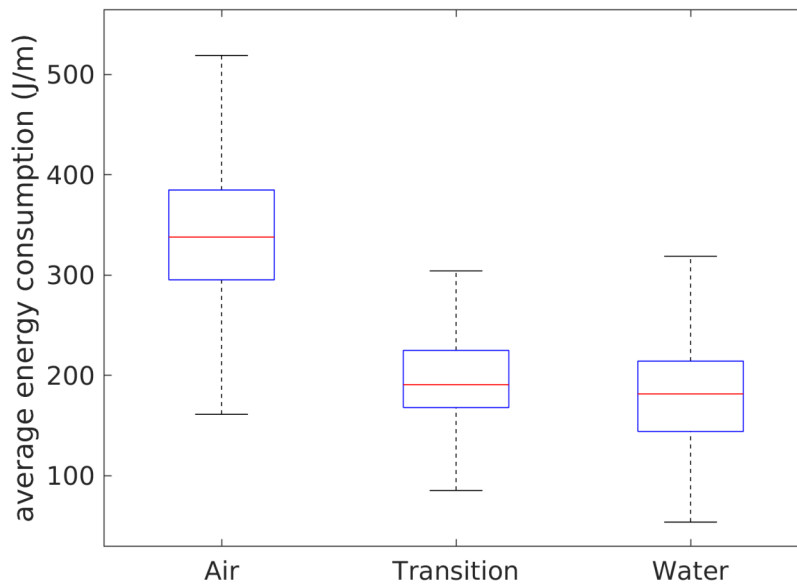


Figure 17: The energy consumption distribution boxplots for each medium.



presented in Table 6.

During the tests, the water-biased approach had advantages in some situations due to its bias towards the water. As verified in a more thorough analysis, it allowed the vehicle to avoid some aerial obstacles that the other two planners struggled with. This led to a higher number of successful missions for the Water-biased CL-RRT, followed by the Estimated Costs Hybrid CL-RRT and the standard Hybrid CL-RRT.

Table 6: Successful missions by planning algorithm out of 200.

Planner algorithm	Successful missions
Hybrid CL-RRT	152(76%)
Water-biased Hybrid CL-RRT	158(79%)
Estimated Costs Hybrid CL-RRT	154(77%)

As mentioned, some scenarios were challenging due to their initial conditions (equally difficult), whereas others were more difficult due to chosen paths (individually difficult) and maneuvering limitations. This led to a difference between the scenarios that were completed by each of the planners. Therefore, in total, 135 (67.5%) scenarios were completed by all three algorithms.

Despite the decrease in the number of scenarios, this experiment and the randomized collection of results were developed with a focus on randomness and the capability to compare a significant number of outcomes. Such characteristics were sufficiently preserved to enable the results to be analyzed. It is also understood that in more conservative application situations, with more safety-friendly navigation algorithms and vehicle models, significantly better results could be obtained.

Therefore, the results for the use of the three planners in the 135 scenarios are presented as follows. Figure 18 presents the data for average energy consumption by each of the algorithms. According to this graph, it can be observed that the original algorithm shows a larger dispersion of values. In contrast, the hybrid planning algorithms present values that are more grouped and apparently a little smaller than those of the original algorithm. Still, the estimated cost heuristic seems to produce similarly grouped values, but with a slightly smaller average than the water-bias heuristic.

Another graph illustrating the characteristics of each algorithm is shown in Figure 19. In this one, three boxplots are presented, each with a notch in its box. This type of graph provides a little more information. The middle line of the box represents the median of the graph; the lower and upper lines of the box represent the first and third quartile, respectively; the solid lines at the ends of the dashed line, represent the tenth percentile and ninetieth percentile, below and above, respectively; and the notch, represents the 95% confidence interval for the median.

Through the graph in Figure 19 it is possible to confirm the statements on the previous graph, such as the statement that the original algorithm produced a larger sample interval for the average energy consumption in the considered scenarios. In addition to this, it can be seen that despite being similar, the distributions for the heuristics present significant differences, since the upper bound for the confidence interval for the median of the distribution of the estimated cost heuristic is 258.69 J/m and the lower bound for the confidence interval for the median of the distribution of the water bias heuristic is 266.31 J/m.

Figure 18: Average energy consumption by planning algorithm. The vertical axis shows the number of samples in the interval represented by the respective blue bar.

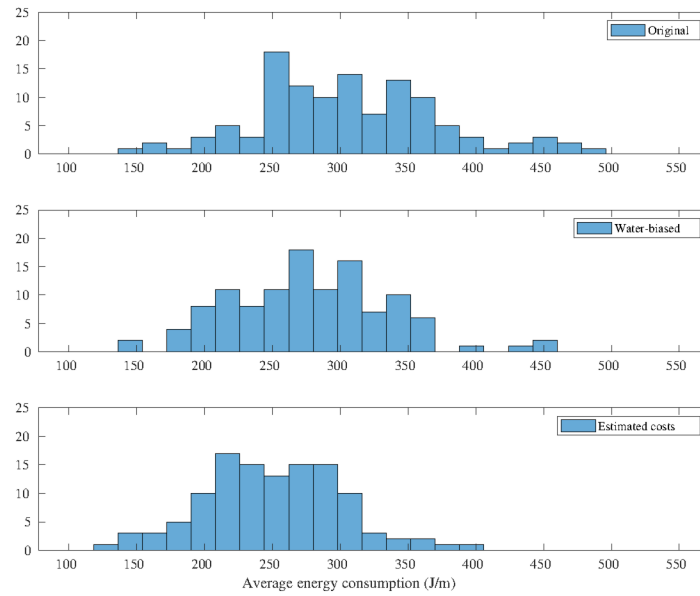
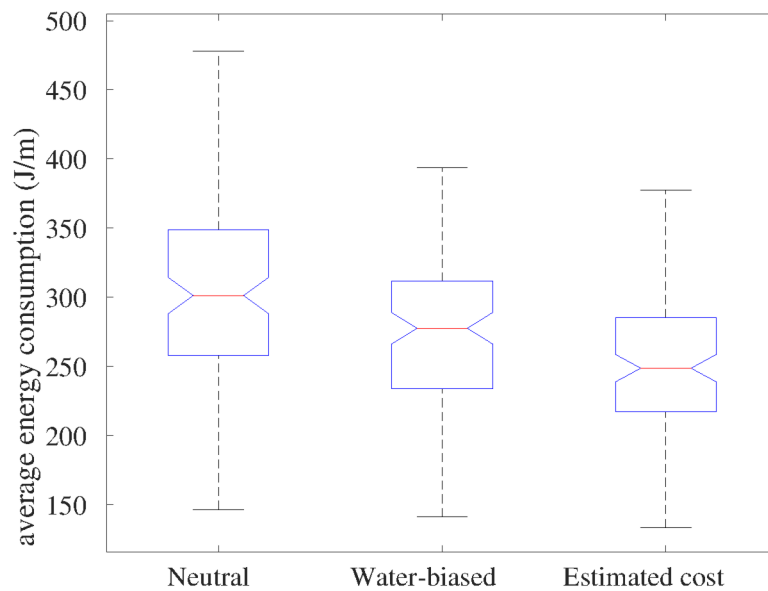


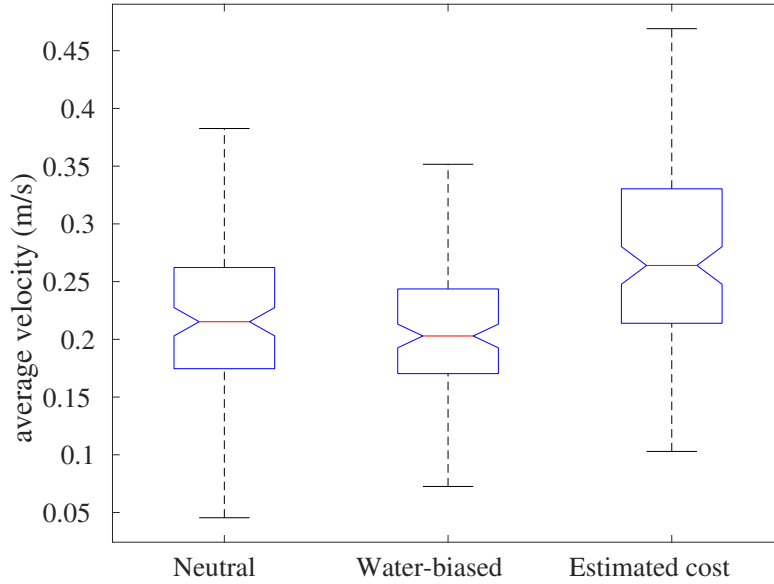
Figure 19: Average energy consumption by planning algorithm.



In addition to the results for energy consumption, results for the average velocity for each algorithm were also surveyed. These are presented in Figure 20. As illustrated in the figure, the water-bias heuristic produced similar average speeds to the original algorithm, while the approximate cost heuristic was able to generate significantly higher results than

the others.

Figure 20: Average velocity by planning algorithm.



Finally, a last perspective is the comparison of the algorithms for each scenario. In this situation, the performance of each algorithm in relation to the others is separately assessed for each one of the cases. Such results are presented in Table 7.

Table 7: Comparison between the energy consumption data of each algorithm for each scenario. The number of times each algorithm was the best (lowest consumption) or the worst (highest consumption) among the three for the 135 scenarios considered.

Planner algorithm	Best	Worst
Hybrid CL-RRT	7 (5.18%)	82 (60.74%)
Water-biased Hybrid CL-RRT	39 (28.89%)	47 (34.81%)
Estimated Costs Hybrid CL-RRT	89 (65.93%)	6 (4.44%)

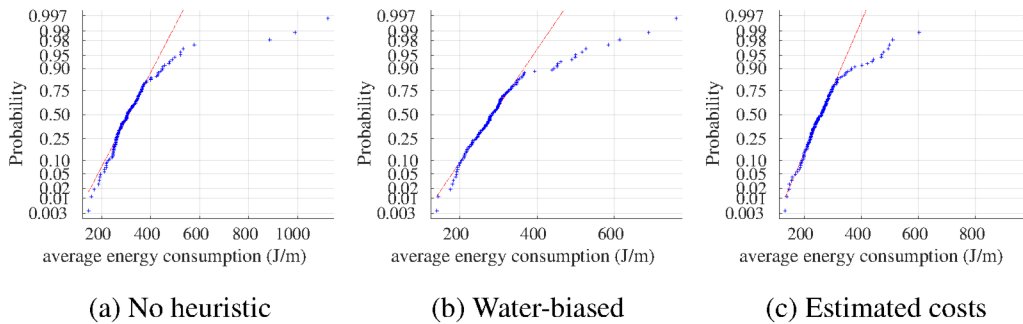
4.4 Results analysis

In this section, the results that were previously presented will be explored. As mentioned previously, a total of 200 tests involving three distinct planning algorithms were conducted. Out of these 200 tests, in 135 instances, all the planners successfully generated safe trajectories for the navigation system, leading to arrival at the objective of the given scenario. Therefore, the analysis presented here is based on these 135 scenarios.

4.4.1 Energy consumption results

The most significant results for this work and the main motivation for proposing the planning heuristics are concerned with the energy consumption by HUAUVs. Since this is a statistical analysis of the performance of these algorithms, it is started by presenting the normal probability plots for the collected energy consumption data. These are presented in Figures 21a, 21b and 21c.

Figure 21: Normal probability plot collected average energy consumption data.



These plots show that the data collected is close to normal distributions. Therefore, they can be analyzed by testing the significance of the null hypothesis. Using the Matlab tool, it was possible to perform a one-sided *ttest* for paired samples. With this, it is concluded that for a significance level of 5%, both heuristics can be considered different and better from the point of view of energy saving.

As expected, the planners developed with the proposed heuristics were able to produce trajectories of lower energy cost for the same scenario when compared to the originally proposed planner (PINHEIRO et al., 2022). This was possible because they use relatively simple physical principles, usually not considered by other works. Probably because these often consider a single domain, such as air or underwater.

The first of these principles consists in biasing the vehicle so that navigates through the medium that requires less energy to move. In this case, the aquatic environment was chosen, as it requires less force for the vehicle to withstand its buoyancy than to sustain its weight in the air. Still, for the scenarios treated in this work, the influence of water currents was not investigated, but these would probably significantly increase the challenges for the maintenance of the results obtained, especially for the planner based on this heuristic.

Another limitation posed by this heuristic is the significantly less exploration of the aerial environment. For hybrid but equally distributed scenarios, such as those treated in this paper, this heuristic was able to present good results. However, in scenarios with a higher number of obstacles in the water, this heuristic would have more difficulties to find feasible trajectories and low energy consumption.

The second principle used for the heuristics was to estimate the cost of the path as it is planned. Once the average consumption for navigation in certain domains is known,

it is possible to estimate the energy consumption for normal operating situations. In this way, it is possible to find the paths that have the lowest accumulated cost and that lead to the goal. However, an extra step was added to this heuristic. Considering that the system can perform all the necessary steps for real-time operation, this planner calculates the realignment of paths according to their costs.

Possibly, the most significant demand of this heuristic is modeling the estimated costs consistently and sufficiently close to the performance in a real scenario. For a real vehicle, the quality of cost estimation would also be affected by atmospheric and water conditions, as well as by the actual actuation system of the vehicle. Thus, large variations in these from what was modeled could impact significantly the obtained results, i.e., the reduction of energy consumption. Path realignment policies have already been presented for RRTs, however, these also demand specific adjustments and would possibly have to be altered for specific HUAUVs and applications.

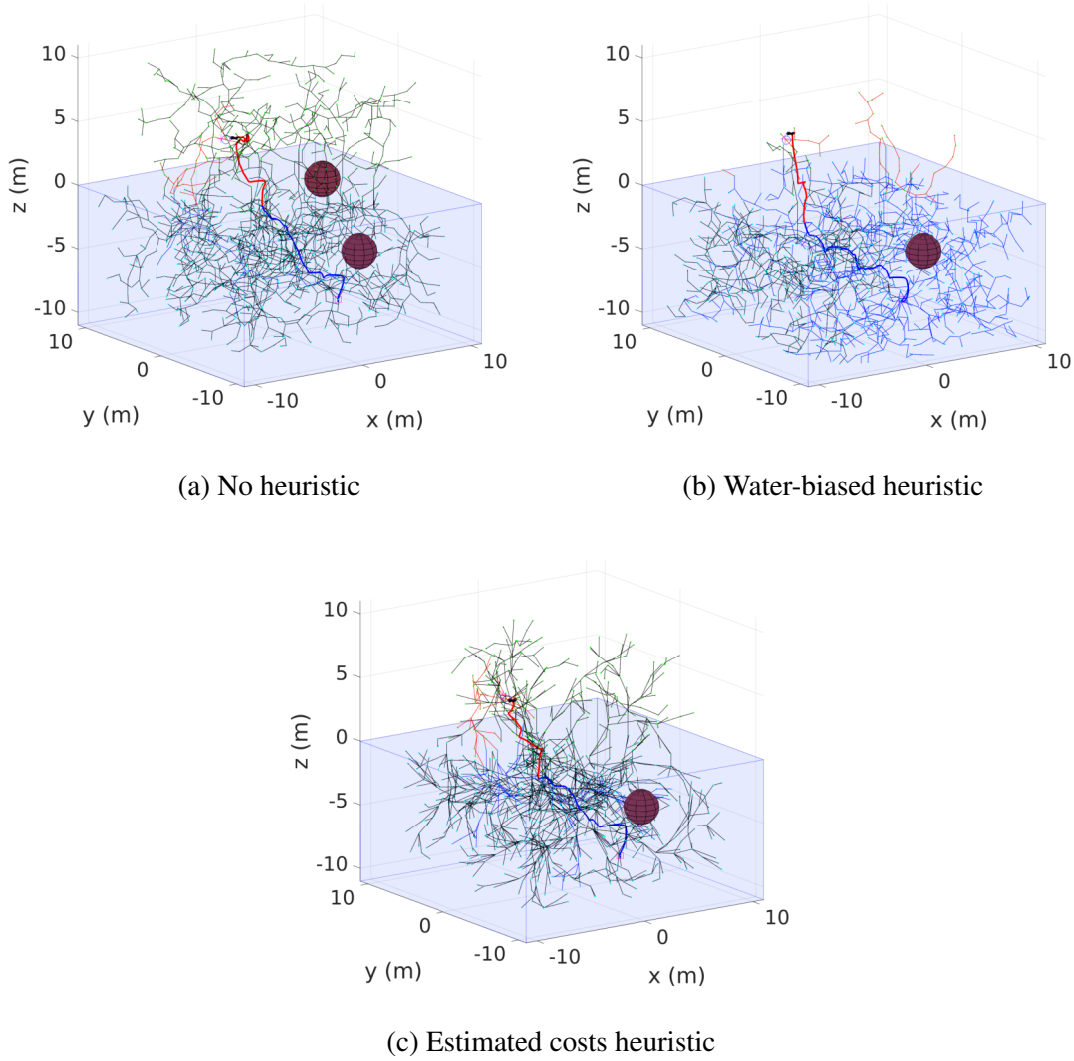
Then, as many factors involved in HUAUVs motion planning still need to be investigated, the simplifications for the adopted scenarios, although representing ideal operating conditions, can be understood as sufficient for the verification of this type of planning algorithm, as well as for demonstrating its potential application for this class of vehicles.

Still, until more simplified models are proved sufficiently close to the dynamics of these vehicles, the propagation of predictive models continues to be costly for real-time systems, especially those that perform some kind of trajectory optimization. Therefore, the heuristics proposed here have the potential to generate trajectories with less energy cost for applications in conditions minimally similar to those presented here, being able to be refined for slightly more complex situations. As presented, these can use simple assumptions of lower computational cost, such as water bias heuristics, or higher cost, such as estimated costs and realignment heuristics.

An example of this difference in the performance of the algorithms can be seen in Figures 22a, 22b, and 22c. These figures show the simulation environment with the vehicle, the detected obstacles (dark red spheres), the underwater trajectory (in blue), and the aerial trajectory (in red). Also, in the RRT, the red edges represent those that are in the air, and the blue ones, those that are in the water. The black edges represent paths excluded from the selection of best nodes because they are branches of nodes that were already visited in the tree.

In all figures, the vehicle describes similar paths. However, it is possible to observe that in Figure 22a the Hybrid CL-RRT algorithm produces inputs for the navigation system that will cause it to change direction more times than in Figures 22b and 22c when navigating in the air. This behavior is characteristic of RRT and corresponds to its random exploration. As in the Water-biased Hybrid CL-RRT, the tree explores the underwater environment significantly more, and when performing the media transition and the beginning of the air navigation, the vehicle follows one of the few paths found toward the goal,

Figure 22: RRT edges (black) and trajectory curve (blue and red) for experiment 32.



which is derived without many changes of directions and leads the vehicle more directly to the goal. And in the Estimated Costs Hybrid CL-RRT, the vehicle travels in a smoother trajectory that leads to a fast domain transition and a goal reached in less time.

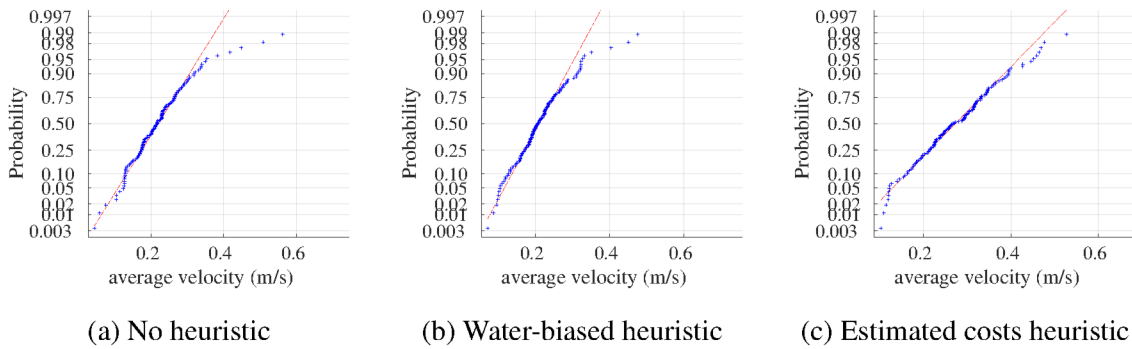
Having a lower energy consumption apparently is related to two factors: traveling in a lower-cost domain and planning for a smoother, more direct trajectory. Traveling in a lower-cost medium may seem a straightforward solution, however, as explained for the aerial trajectory of the Hybrid CL-RRT algorithm, changing directions may waste vehicle momentum and contribute to higher overall energy consumption. Additionally, since the map is unknown, only planning for smooth, direct trajectories, regardless of the medium, may lead to detecting obstacles in the planned trajectory while not in an energy-friendly medium, and possibly leading to a higher energy consumption.

4.4.2 Velocity results

As presented in Section 4.3, the CL-RRT and Water-biased Hybrid CL-RRT algorithms had similar behaviors from the point of view of the average velocities between the start and end points of their paths. The Estimated Costs Hybrid CL-RRT algorithm presented higher average velocities than the other two algorithms. Thus, this algorithm can be classified as the best for this set of tests, since it significantly outperformed the others, both in energy consumption reduction and average speed increase. Both qualities are usually sought in missions with HUAUVs.

And as for the statistical analysis of the performance of these algorithms, the following normal probability plots for the collected average velocity data are presented in Figure 23.

Figure 23: Normal probability plot for velocity data.

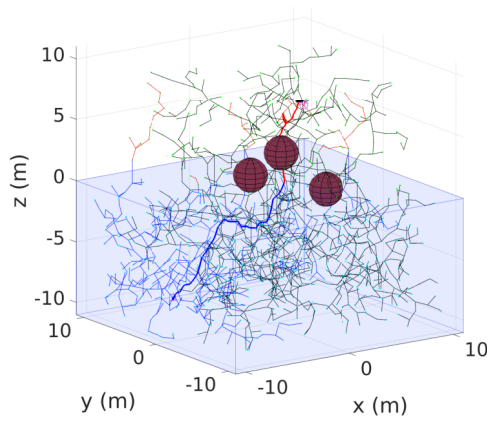


These plots show that the data collected is close to normal distributions. Therefore, they can be analyzed by testing the significance of the null hypothesis, the same as in the previous section. Through it, it can be concluded that for the significance level of 5%, both the CL-RRT and Water-biased Hybrid CL-RRT algorithms can be considered to produce lower average velocities when compared to the Estimated Costs Hybrid CL-RRT algorithm. The performances of the planners concerning average speed can be better understood by analyzing Figure 24.

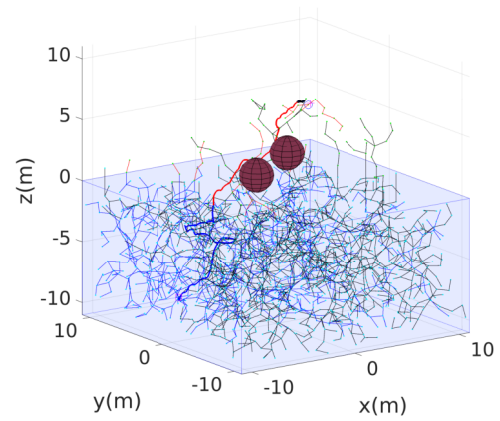
Figure 24a presents the RRT for the hybrid planning algorithm without heuristics. Through this it is possible to observe that during the mission time, the RRT explored similarly the aquatic and aerial environments, inserting new nodes through $2m$ edges at each iteration. As is typical of RRTs, these edges change direction significantly relative to the parent node edge, producing a meandering tree. This behavior makes it difficult for the underwater dynamic model of the vehicle to follow, so the vehicle ends up taking longer to reach the position references.

As for Figure 24b, it can be observed that some characteristics of RRT for the previous algorithm can be observed. However, for the hybrid planning algorithm that uses the water bias heuristic, the underwater environment, as proposed, is explored significantly more by RRT. Thus, as much as the vehicle navigates for longer in an environment that consumes

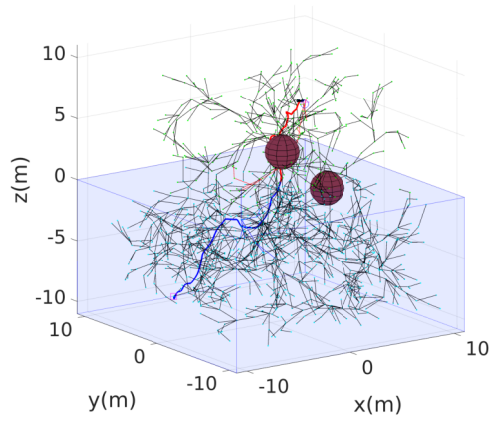
Figure 24: RRT edges (black) and trajectory curve (blue and red) for experiment 11.



(a) No heuristic



(b) Water-biased heuristic



(c) Estimated costs heuristic

less energy for its displacement, it also ends up being subjected to a condition of less maneuverability for longer as well. This results in a lower average speed to reach the goal.

Finally, for Figure 24c, different characteristics can be observed for the RRT that uses the heuristic of estimated costs and node rewiring. This produces paths that change direction with less intensity, as proposed by Karaman and Frazzoli (2010). Thus, the vehicle can follow the references passed in a smoother way and ends up reaching the goal in a more direct and faster way.

4.5 Results considerations

The presented results have yielded valuable insights into the behavior of multidomain vehicle planning. Initially, it was expected that planning based on geometric exploration would result in inefficient references for the model predictive trajectory generator. However, what was not foreseen was the significant impact of employing heuristics to modify planner properties.

Both heuristics were designed based on a fundamental principle of energy conservation: achieving motion with minimal effort. One approach involved estimating energy consumption along a specific route, while the other biased the trajectory towards the lower-cost domain.

What emerged as unexpected was that considering the estimated energy consumption of a route not only led to energy savings but also resulted in faster-executed trajectories. It is important to note that these findings are strongly influenced by the specific characteristics of the simulated scenario. Therefore, it is essential to view these conclusions as preliminary rather than definitive.

In conclusion, the presented results provide strong evidence for the potential application of the planning techniques in computing trajectories for multidomain HUAUVs. The diverse range of scenarios examined demonstrates that, in this problem formulation, the results obtained are highly significant and capable of generating lower-cost trajectories. The findings highlight the effectiveness and practicality of these techniques, particularly considering their algorithmic simplicity. Further research and exploration are needed to fully validate and optimize their performance. Nevertheless, these findings offer promising insights for the adoption of these techniques in low-cost autonomous platform motion systems.

5 CONCLUSION

This work aimed to address the growing demand for automation in hybrid tasks, particularly those involving partially submerged objects of interest. It was believed that hybrid unmanned aerial-underwater vehicles could effectively fulfill these tasks. Over the past two decades, the field of study surrounding these vehicles has seen significant development. Nevertheless, there are still certain key elements lacking in the pursuit of robust autonomous systems.

This study also covered a comprehensive review of the literature associated with this class of vehicles, providing a brief overview of the planning approaches employed. It became evident that the majority of works in the current state of the art regarding HUAUVs do not adequately address the planning problem. However, it was demonstrated that there are numerous powerful techniques available for tackling this challenge. Furthermore, it was suggested that combining these techniques has the potential to yield significant results.

Additionally, this study presented the vehicle dynamics and the energy consumption model, which serve as essential inputs for the proposed planning algorithms. Subsequently, the planning algorithm itself was elaborated in detail, along with the introduction of two proposed heuristic variations. Pseudo-code was provided for each implementation to facilitate understanding. Finally, some key performance metrics were presented to assess the effectiveness of the proposed approaches.

The first (standard) algorithm was based on the one proposed by Kuwata et al. (2009) and adapted for the hybrid environment and proved to be as robust as its application for single-domain vehicles. However, this one did not have any form of optimization or energy consumption reduction heuristics, unlike the other two proposed algorithms.

The second proposed algorithm used a simple heuristic of bias towards the environment that would naturally demand less energy for the displacement of the vehicle. Through this, it was possible to intuitively improve the vehicle's energy consumption in simulation.

Finally, the third proposed algorithm used a less practical heuristic which required a survey of cost estimates for each medium. Furthermore, to increase its efficiency, the

technique of node rewiring was used to locally improve the RRT employed. In this way, this algorithm was the best among the three, considering ideal operation conditions as those used in the experiments.

Based on the aforementioned findings, it can be concluded that this work successfully achieved all of its objectives. Specifically, the mathematical model was modified to accommodate a smoother and more realistic domain transition. Moreover, a straightforward simulated scenario was created, enabling the examination considering obstacles and the dynamics of the hybrid domain. Additionally, a base trajectory planning algorithm was formulated, and two heuristics were introduced. Furthermore, a comprehensive comparison was conducted between the standard algorithm and the proposed heuristics, and the resulting outcomes were thoroughly presented.

5.1 Future work

Throughout this work, many interesting situations were identified that could be studied. However, due to time and resource constraints, these represent opportunities for future work.

One of these opportunities is related to the random scenarios used to compose the results not considering atmospheric or underwater currents. These would possibly alter the results found for the velocities and for the energy consumption of these algorithms. Therefore, it could be studied if these algorithms would need to be adjusted for more variable conditions and if these adjustments would be enough to maintain their good performance.

Another opportunity that was not investigated in this work refers to the use of the technique of node rewiring, used in the estimated cost algorithm. This technique could be employed in the other algorithms, and possibly contribute to more direct and agile trajectories. Still, as mentioned in different points of this work, the use of this technique together with the planning algorithms could be applied in real systems in experiments in controlled environments to verify its ability to run in real-time HUAUV systems.

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